PHYSICAL MEASUREMENT.

A

COURSE OF EXPERIMENTS

IN

PHYSICAL MEASUREMENT.

In Four Parts.

PART IV.

APPENDIX FOR THE USE OF TEACHERS.

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APPENDICES AND EXAMPLES FOR THE USE OF TEACHERS.

APPENDIX I.

THE LABORATORY.

THE first requisite for a course in elementary Physical Measurement is a well lighted and uniformly heated room, with the ordinary precautions to secure good ventilation. Experience has shown that these advantages cannot practically be obtained in basements, however suitable the latter may be for certain scientific purposes. The first floor of a building, properly supported by brick pillars to prevent vibration, has undoubted advantages for a course of measurements. The use of iron in construction should be in so far as possible avoided on account of its magnetic influence.

If the room above a physical laboratory is to be occupied, there should be an empty space between the floor of that room and the laboratory ceiling.

The latter should *not* be supported by a rod or rods connecting it with the former, but by separate beams or trusses reaching to the walls. Under these conditions only will the vibrations of the upper floor be cut off.

Sometimes, to avoid annoyance from this source, the laboratory is placed in the upper story of a building. The advantages of skylights as a method of illumination, and rafters for purposes of suspension, have been justly urged. They are, however, offset by many practical objections, among which may be mentioned the danger of leakage and the accumulation of dust, both occurring at inconvenient altitudes. Rafters are, moreover, not good points of suspension, since the roof of a building is very sensitive to the wind, and to other sources of vibration. For these reasons, and on account of economy in heating, attics are undesirable for the purposes of physical measurement.

The best possible place in a large building for laboratory work is that usually set aside for lecture purposes; namely, a two or three story room reaching from the first floor to the attic floor, and situated either in an L, or at one end of the building, so as to be lighted from three sides. The attic should be used solely for the storage of apparatus, or as a means of reaching different points in the laboratory ceiling, where suspensions, for instance, may be needed. In the absence of a two-story room, a staircase may perhaps be utilized for long suspensions when necessary (Exp. 65).

The laboratory should be situated as far as convenient from public travel, but not too far from the work-shop. It is hardly desirable that a powerful engine of any sort should be placed under the same roof as the laboratory. Even a smoothly running gas engine may interfere with delicate measurements. The neighborhood of dynamo and even telegraph wires should be avoided, and particularly vertical portions of such wires. Powerful currents, if admitted at all to the laboratory, should come and go through parallel wires (see ¶ 193, 8).

The conditions here named are not so formidable as they may perhaps appear. A common square wooden building with an attic and basement, situated in the middle of a field, has advantages for the purpose of physical measurement which some of our best institutions do not possess.

It is well to have the laboratory ventilated by registers in the floor and ceiling. A large number of small registers in the floor is likely to cause less draught than one or two large registers. It is best, on account of dust, to heat the air before it enters the room, by steam. Furnace heat is, however, not objectionable if the pipes, furnace-chamber, and air-boxes are perfectly tight, so that no dust from the ashes can enter them. The floor, walls, and ceiling, should be finished in paint, wax, oil, shellac, or varnish, so that they may easily be kept clean. Other precautions against dust will be spoken of later on.

The window-sills, if of the ordinary pattern, should be at least 2 ft. 10 in. from the floor, in order that

work-tables or benches may be placed in front of them without cutting off the light. There should be, moreover, no obstacle — such as steam-pipes — to prevent such tables or benches from being set close to the walls. This is not only a matter of convenience in preventing small objects from falling behind the tables, but also in some cases the only means of making tables steady enough for delicate experiments. In some cases, such tables have to be supported by pillars reaching to the basement; but this will not be necessary for elementary work.

It is well, on at least one side of the room, not exposed to the sun, to have a wooden bench, 2 feet wide, 2 in. thick, and 33 in. high, fitted permanently to the wall, and if necessary, into the window spaces. A small platform or balcony facing southward will be found convenient for experiments requiring *direct sunlight.*

The windows on the sunny sides of the room should have blinds giving free access to air. White window curtains, closing from below upward, are a convenience, but by no means a necessity. Wooden shutters, or curtains of enamelled cloth almost entirely opaque to light, are needed on all the windows to obtain the best results in photometry, and are absolutely indispensable if experiments in photography (not included in this course) are to be made.[1]

The expense of arranging a laboratory so that it may be darkened is very much less than that of con-

[1] For the latter purpose, it would be well to have yellow glass set into one or two of the windows.

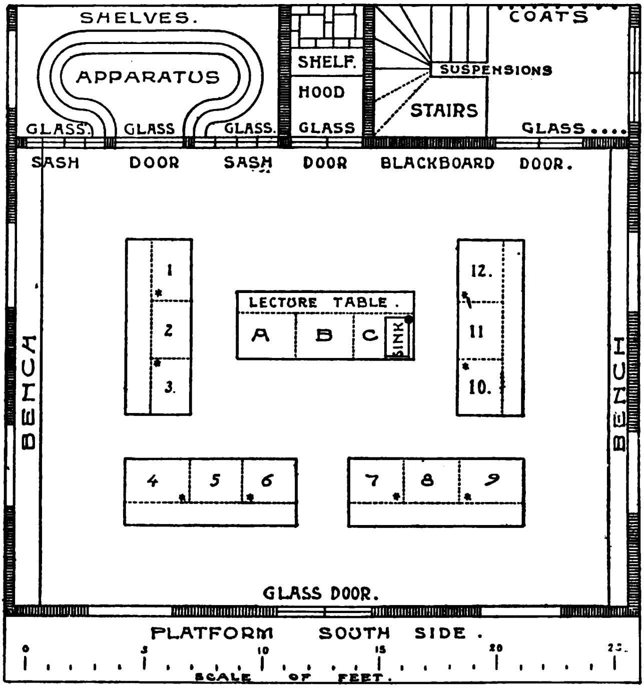
structing the customary "dark room" for photometric and photographic experiments, and has the advantage of furnishing plenty of air and space to the students. This arrangement is, however, practicable only when, as in the course of experiments which the author has planned, *all* the students are to work together at a given time upon a given class of experiments.

There is no need, under these circumstances, of having a separate "weighing room" or a separate room for electrical measurements. Ordinary ballances and galvanometers, provided with glass cases, may generally be left in place without danger of injury. It may occasionally be desirable, during experiments with certain corrosive acids, to shut up all the finer apparatus in a cabinet, and for this reason such a cabinet should be provided. The cabinet should have glass sides and doors if possible, and plenty of shelves. It should occupy about $\frac{1}{20}$ as much floor space as is necessary altogether for the accommodation of students (see plan, page 906).

The furnace flue should be built side by side with one or more ventilating flues in a chimney next one wall of the room. A small closet or "hood" should be built round the chimney (see plan). This hood is intended as a place to store batteries and chemicals from which noxious odors may arise. There should be a continual current of air passing into it from the room, and out through an opening near the top into one of the ventilating flues. It is convenient, but not necessary, to make this closet large enough to

work in. A large wooden box placed in front of an open fireplace is an excellent substitute for a hood.

There will be needed for lecture purposes, or for purposes of demonstration, a table of considerable size, about 34 in. high, not too far from one of the

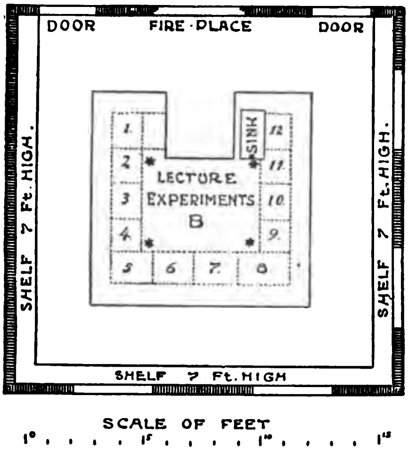


PLAN OF LABORATORY FOR SECTIONS OF 12 STUDENTS EACH.

walls where a blackboard may be placed, and in a position where it may be seen from all parts of the room (see plan). This table should be supplied with water, drainage, and gas, which can, in the absence of electricity, be used for both heating and lighting pur-

poses. If steam is used in the building, a steampipe leading to the table may also be a convenience. The table should be furnished with drawers and cupboards, in which the lecturer may keep the greater part of the instruments which he will need during the course (see plan, A, B, and C, page 906).

Other tables, 30–34 in. high,[1] also furnished with drawers and cupboards for the use of students (see plan, 1–12) may be conveniently arranged in a hollow



Plan of Laboratory for Sections of six Students each.

square, so that the students working at them may face the lecturer. If the room is very small, all these tables may be built into one. The author has made use of a table (Fig. 2) 9 × 9 ft. square in lecturing to a class of 10 or 12 students, and in directing the

[1] 30 inches if chairs are to be used by the students, 34 inches if the students are to work standing, or to make use of high stools.

work of at least 6 students at one time. Around the outside of the table were 12 drawers with small closets beneath them, so that each student had a special place for apparatus. Some such arrangement is necessary, to bring into play in the minds of students that sense of at least temporary ownership which lends interest to the study, as well as to the preservation of the instruments intrusted to them.

In addition to the table or tables already mentioned, a considerable number of common tables or benches may (if space permits) be utilized for instruments which are to remain permanently in place; as, for instance, ordinary balances (Exp. 6), optical benches (Exp. 41), electric micrometers (Exp. 65), astatic galvanometers (Exp. 76), and "dip" apparatus (Exp. 77). These details will depend largely upon the system adopted for carrying on the course.

APPENDIX II.

APPARATUS.

THE apparatus which will be required for the course of 100 experiments described in this book, and for a few outside experiments needed for the purpose of illustration, is catalogued below under 20 heads, following the order in which the apparatus is wanted. The author has selected in the case of alternative methods, such apparatus as he himself has found most convenient to employ.

1. **For general use,** — first, measures of length: (*a*) a *Metre Rod*, graduated in millimetres either on wood, brass, or steel. Wooden rods have the smallest coefficient of expansion, are satisfactory, and cost only about 25 cts.; (*b*) a *Measure* graduated in *cm.* on cloth or steel tape 10 metres long (Fig. 224), cloth, costing about 75 cts.; (*c*) a *Gauge* (with vernier), reading to $\frac{1}{10}$ *mm.*, with a shaft 10 or 15 *cm.* long, made to order for about $1.00 in Paris (Fig. 2); and (*d*) a *Long Gauge* with vernier reading to $\frac{1}{10}$ *mm.*, with a shaft 40 or 50 *cm.* long (Fig. 222), made to order for about $1.50 in Paris; one of these is enough for 5 or 10 students. Other instruments for the measurement of length will be found under 5.

Second, for the measurement of weight: (*e*) a *Balance* (20 *kilo* capacity) with weights (Fig. 188), costing \$5.00 or \$10.00; one instrument enough for 5 or 10 students; (*f*) a *Balance* (flat pan) sensitive to $\frac{1}{10}$ gram (Fig. 1), costing \$3.00 or \$4.00; (*g*) *Weights* (iron) for ditto, cost about 50 cts.; (*cg.*) *Weights* (1 *cg.* to 100 *g.*), costing from \$2.00 to \$5.00 (*c*, Fig. 4). Other instruments for the measurement of weight will be found under 3.

Third, for the measurement of time, (*h*) a *Clock* with a wooden seconds pendulum (Fig. 152), to which a break-circuit may be attached, so as to re-enforce the ticks by electrical means. The second-hand can be made also to close a circuit, so as to give the signals (needed for the determination of rates of cooling) once or twice a minute. Such a clock (without connections), made by the Seth Thomas Co., costs about \$20.00. One is of course enough for all students.

Fourth, (*i*) a *Barometer* (aneroid), costing \$5.00 or \$10.00 (4, Fig. 53); one for the whole class; (*j*) one *Hygrodeik*[1] (Fig. 13), for the whole class, \$5.00 or \$10.00; and (*k*) a *Thermometer* (Fig. 51), 0° to 100° Centigrade, graduated on glass stem, costing about \$1.00 in Europe, and \$3.00 in the United States, — if possible, one thermometer for each student.

Finally, (*l*) a *Lens* or magnifying glass (Fig. 34), costing about 50 cts.; (*m*) a spirit *Level*, costing

[1] A substitute for a hygrodeik consists of a pair of thermometers, one of which has its bulb surrounded with wet lamp-wicking. The readings of these thermometers are to be interpreted by Table 15.

about the same; and (*n*) some wooden *Blocks*, on which to mount apparatus.

Total cost, $40.00 to $60.00.

2. **Preliminary Experiments** (I. to IV.): (*a*) a wooden *Block* (about 10 *cm.* cube); (*b*) a similar *Block hollowed* with an auger 5 *cm.* in diameter to a depth of 8 *cm.*, then closed by a wooden plug even with one surface; (*c*) a *small Block* about 3 *cm.* cube, — all of these blocks oiled or paraffined to make them impervious to water; (*d*) 10 or 12 *steel Balls*, such as are used in the bearings of a bicycle wheel, weighing about 30 grams (or as much as a hydrometer can float), and costing about $1.00; (*e*) a *Brush* of camel's-hair; (*f*) and a *Nicholson's Hydrometer* (Fig. 4, page 8), costing from $3.00 to $5.00.

A gauge, balance, weights, thermometer, &c., have already been mentioned (1, (*c*), (*f*), (*g*), (*cg*), (*k*), &c.). In the rest of this list the names of instruments once mentioned will not in general be repeated.

Total cost, $5.00 to $7.00.

3. **The Balance** (Exps. V. to X.): (*a*) a *Balance*, sensitive to 1 *cg.*, either uncovered as in Fig. 14, page 26, or with a glass case (Fig. 15, page 28), and costing from $6.00 to $22.00. The capacity of the balance should be at least 100 grams; (*b*) a *Barodeik* (Fig. 14, page 26) consisting of a litre flask hermetically sealed, and counterpoised by pieces of window glass. Such a flask should cost not over $1.00. It is well to have one such flask and counterpoise permanently mounted on a special balance (sensitive to 1 *cg.*) with a paper scale especially constructed so as

to show the density of the air. The complete instrument ought not to cost more than $10.00; (*c*) a *Barometer tube* (Fig. 10, page 17), 80 *cm.* long, 5 *mm.* internal and 10 *mm.* external diameter (cost 25 to 50 cts.); (*d*) a *small Beaker* (*b*, Fig. 10), costing less than 25 cts.; (*e*) a *nickel-plated Cup*, holding 250 grams or more (cost 25 or 50 cts.); (*f*) a *standard Weight*, (100 *g.*), costing about $1.00; (*g*) a *glass Ball* (or marble) 4 or 5 *cm.* in diameter (about 10 cts.); (*h*) 2 *small Rings* of equal weight (to prevent ball from rolling); (*i*) a hydrostatic *Arch* (Fig. 18, page 43), made of sheet brass for less than 25 cts.

Total cost, $10.00 to $35.00.

4. **The Specific Gravity Bottle, &c.** (Exps. XI. to XVIII.): (*a*) a *Specific Gravity Bottle*, or a common "2 ounce," wide-mouth, glass-stoppered bottle (Fig. 19, page 50). The stopper should be solid so as not to enclose a bubble of air. If hollow it must be filled with paraffine or other material not acted upon by ordinary liquids; cost less than 25 cts.; (*b*) a *Densimeter* with jar (Fig. 20, page 60), costing about $2.00; (*c*) a *U Tube* with rubber couplings, and (*d*) a *Y Tube* with rubber couplings, both tubes of glass or metal, about 3 *mm.* internal and 7 *mm.* external diameter (cost about 25 cts.); (*e*) 4 straight *glass Tubes* of the same diameter and about one metre long (about $1.00); (*f*) a *rubber Tube* (for connections), 1 metre long, 6 *mm.* internal diameter (25 to 50 cts.); (*g*) an *Air Pump*, or Richards' injector; the latter can be attached to an ordinary water faucet, and gives a more or less perfect vacuum; cost about $1.00; (*h*) a small *stop-cock*

ending in 2 tubes 6 or 7 *mm.* outside diam. (about 25 cts.); (*i*) a stout *Flask* (Fig. 24, page 67), capable of resisting the atmospheric pressure (25 cts. to $1.00); and (*j*) 3 *India-rubber Stoppers* to fit the flask, with 2 holes, with 1 hole, and with no hole (cost about 10 cts. each).

Total cost, $5.00 to $7.00.

5. **Length** (Exps. XIX. to XXI.): (*a*) a *Micrometer Gauge* (Fig. 28, page 73), reading to $\frac{1}{100}$ *mm.*, made to order for about $2.00 in Paris (American instruments reading only to $\frac{1}{50}$ *mm.* cost from $4.00 to $6.00); a *Spherometer* (Fig. 38, page 83), costing about $20.00. A much cheaper instrument, accurate enough for most purposes, could undoubtedly be made by soldering the necessary appendages to a nut and screw with a millimetre thread. Such threads and instruments for cutting them, though rare in America, are common in France. (*c*) A piece of *plate Glass* about 5 *cm.* square (10 cts.). The Vernier Gauge, Balls, and Lens, have been mentioned in 1, 2, and 3.

Total cost, $5.00 to $25.00.

6 **Expansion** (Exps. XXII. to XXX.): First, thermometers of the ordinary sort (see 1, *k*), at least one for each student; then (*a*) an *Air Thermometer* (or manometer) consisting of a stout glass tube 40 *cm.* long, 2 *mm.* internal diameter, graduated in *mm.* (see Fig. 56, page 119), at a cost of about $2.00; (*b*) for purposes of illustration (only), an *air-pressure Thermometer* (Fig. 60, page 127). The bulbs *a* and *c* should be about 5 *cm.* in diameter, and should have

capacities of at least 100 and 200 *cu. cm.* respectively. The tube *b* should be graduated in *mm.* for a distance of 40 *cm.* or more, and should have an internal diameter of about 2 *mm.* A similar instrument (made by Alvergniat, Paris) cost $3.00 unmounted. Add for mounting, and for a kilogram of mercury, about $2.00. One instrument enough for class. Corrections for different readings are easily calculated, and should not exceed 1° ; (*c*) a *weight Thermometer*, or test-tube drawn out to a fine point (see ¶ 240), and (*d*), a *self-registering Thermometer* (or maximum and minimum), costing $3.00 in London. One instrument enough for a class.

Next, for heating (or cooling) purposes: (*e*) a *Bunsen burner* ($1.00) (3, Fig. 53), or its equivalent; (*f*) a *steam Boiler* (Figs. 53 and 54, page 115), capable of admitting a thermometer ($1.50); (*g*) a *rubber Tube* for steam (*ad*, Fig. 46, page 90), 50 *cm.* long (about 25 cts.); (*h*) a *Steam Jacket* or tube (*di*, Fig. 56) 1 metre long, and 3 or 4 *cm.* in diameter, with corks (50 cts.) ; (*i*) an *ice Trough*, 1 metre long and 5 *cm.* deep, made of a strip of tin, and (*j*) a *vapor Boiler*, or stout flask of about 100 *cu. cm.* capacity, drawn out at the mouth into a tube 5 or 6 *cm.* in diameter (about 25 cts.). It may be well to surround the flask with wire netting in case of accident. [The use of this apparatus is *not* accurately represented in Fig. 64. The manometer should be raised on a block of wood and fastened there, so as not to roll, the rubber tube should slope *toward the boiler*, and the boiler should be nearly covered by the hot water.]

Finally, the special apparatus (*k*) of *Dulong and Petit*, modified as in Fig. 47, page 95 (brass, about \$4.00); one enough for 4 or 5 students, since each may take his own observations; and (*l*) the *Manometric apparatus* of ¶ 76 (Fig. 62), costing about \$1.00, exclusive of the filter-stand, and mercury. It might be well to have two bottles blown especially for this apparatus, with tubes issuing directly from the top and bottom. One instrument enough for 2 students; (*m*) A wooden *Micrometer Frame* (*bcon*, Fig. 46), with screws *f* and *j*, ought not to cost more than \$1.00; (*n*) two *Test tubes* (10 cts.), and (*o*) a *Medicine dropper* (10 cts.; see Fig. 58, page 121), with the flask and stoppers already mentioned (see 4, *i* and *j*), will be required.

Total cost \$15.00 to \$20.00.

7. **Calorimetry** (Exps. XXXI. to XXXVIII.): (*a*) a *Calorimeter* (Figs. 70, 71, 72, page 144), with an inner cup of 100 or 200 *cu. cm.* capacity (50 cts. to \$1.00); (*b*) a *Stirrer* (Fig. 50, page 107); (*c*) a *Measuring glass* (Fig. 75, page 159) of 2 or 3 *cu. cm.* capacity (25 cts.); (*d*) a *Steam Shot heater* (Fig. 79, page 179), with an inner cup of 100 or 200 *cu. cm.* capacity (about \$1.00); (*e*) a *Bottle for ice-water* (wide mouth, 250 *cu. cm.* capacity, with wire netting to restrain ice,—about 10 cts.); (*f*) a *Steam Trap* (*b*, Fig. 83, page 203), materials costing about 25 cts.; (*g*) a glass *Beaker for Calorimetry*, of 100 or 200 *cu. cm.* capacity (see ¶ 105 (1), cost less than 25 cts. Each student should have at least one thermometer (see (1)) at his disposition during these experiments.

Total cost, \$2.00 to \$3.00.

8. **Radiometry** (Exps. XXXIX. to XL.): (*a*) an *Optical Bench* (Figs. 89 and 100), consisting of a board or plank 1 or 2 *m.* long, 10 *cm.* broad, set up edgewise, with 8 grooved blocks or "sliders" to fit loosely over it. One slider is to hold a candle, another a kerosene lamp, a third a telescope, a fourth a lens, a fifth a pasteboard screen, a sixth and seventh wires (Fig. 104), and an eighth a wire netting. Each slider can be clamped by a small screw eye, and each carries a marker. A paper or wooden *mm.* scale is attached to the board. Cost of bench and sliders, \$2.00 or \$3.00; (*b*) a Bunsen *Photometer* (Fig. 94, page 224). The diaphragm (incorrectly represented in *de*, Fig. 94) consists simply of a piece of paper with an oil-spot (Figs. 91–92) mounted as in *de*, Fig. 93,—cost, possibly 50 cts.; (*c*) a *Candle*; (*d*) a small *Kerosene Lamp;* and (*e*) either a *differential Thermometer* with gauge (Fig. 86, page 216), or else a Thermopile and Galvanometer (Fig. 88). Cost of the differential thermometer (made nicely by a tinman), \$1.00 or \$2.00.

Total cost, \$4.00 or \$5.00.

9. **Focal Lengths** (Exps. XL. to XLIII.). In addition to the Optical Bench, &c., mentioned in 8, there will be needed (*a*) a small *Telescope* (with cross-hairs), which may be borrowed from the sextant or spectrometer mentioned below (in 10); (*b*) a *long-focus Lens*, the achromatic object-glass of the telescope; (*c*) a *converging Lens* (the crown glass of the combination) and (*d*) a *diverging Lens* (the flint glass of the combination). These lenses if 3 or 4 *cm.* in

diameter, would cost separately $3.00 or $4.00. Spectacle lenses at less than $1.00 per dozen will answer most purposes. It is desirable to have (*e*) 3 small *Mirrors*, (slightly) convex and concave, for purposes of illustration (¶ 118), and if possible (*f*) a "*Doublet Lens*," such as is used for rectilinear photography, costing about $15.00 (one sufficient for 4 or 5 students). The principle can be illustrated by two "meniscus" spectacle lenses mounted facing each other with a small diaphragm between them.

Total cost $1.00 to $20.00 (the former, if spectacle lenses are employed, or if lenses are borrowed from optical instruments elsewhere accounted for).

10. **Goniometry** (Exps. XLII. to XLVII.): (*a*) a *Sextant*[1] (Fig. 106, page 246) reading to 10″, costing $15.00 in Liverpool; (*b*) a Babinet *Spectrometer*,[1] reading to minutes or fractions of a minute, costing about $20.00 in Paris. A cheap spectrometer can be made with a paper scale, but such instruments are apt in the hands of students to yield unsatisfactory results. It is perhaps better to dispense with the spectrometer altogether, and to use a sextant in its place. In this case a metallic shield with (*c*) a narrow *Slit* (1 *mm.* × 10 *cm.*) will be required in certain experiments. This must be illuminated by (*d*) a *Sodium Flame* (see foot-note, page 260). (*e*) an *Artificial Horizon* (page 545); (*f*) a small *Prism*, and (*g*) a *Diffraction Grating* (page 267), complete, at a nom-

[1] It is indifferent whether the student begins with the sextant or with the spectrometer. One instrument of each kind is therefore enough for at least 2 students.

inal expense, the list of apparatus required for angular measurements.

Total cost, \$35.00 to \$80.00.

11. **Sound** (Exps. XLVIII. to LV.): (*a*) a *Resonance Tube* (Fig. 121, page 272), costing \$1.00 or \$2.00; (*b*) a *Monochord* (Fig. 122, page 274), costing perhaps \$2.00; (*c*) a *Bow* (*ai*, Fig. 122), costing \$1.00; (*d*) *Signalling* apparatus, for instance, two hammers and boards (¶ 137, III.); (*e*) a *Smoked glass* apparatus (Fig. 125, page 288). The dimensions may be as follows: total length 60 *cm.*, breadth 15 *cm.*, height 30 *cm.*, cost about \$3.00 for carpenter's work; (*f*) a *Toothed Wheel* apparatus (Figs. 135 and 136, page 302), watchmaker's charge about \$1.00; (*g*) *Tuning Forks* with the following rates of vibration per second: G♯ = 51.2; A = 54; A♯ = 57; B = 60; C = 64; C = 128; A = 216; C = 256. Cost made by blacksmith out of best steel, about \$8.00 in all. Additional forks (68, 72, &c., up to 128) desirable, but not necessary. Forks A = 440 + and C = 512 + to be had of dealers in musical instruments (25 cts. each); (*h*) a *Pitch Pipe* (Fig. 273, page 554), or its equivalent (less than \$1.00); (*i*) a *Cloth Band* for torsional vibrations (page 554); and (*j*) means of stretching considerable lengths of *Wires*. Resin will be needed in these experiments. Students will do well to work in pairs upon all these experiments.

Total cost, \$15.00 to \$20.00.

12. **Kinetics** (Exps. LVI. to LX.): (*a*) a *simple Pendulum* (Fig. 150, page 316), at a nominal cost; (*b*) an *irrotational Pendulum* (Figs. 153 and 154, page 321,

scale of figures about $\frac{1}{30}$), cost, made by ordinary carpenter, tinman, and blacksmith, not over $5.00; (*c*) a *Torsion Head*, or suspension (AB, Fig. 159 page 334), costing about $1.00; (*d*) a *Torsion Pendulum*, or ring apparatus (BCEFD, Fig. 159), costing about $1.00; (*e*) a *Spiral Spring* apparatus (Fig. 158, page 331), cost nominal; (*f*) a *Falling-Bodies' apparatus* (Fig. 149, page 313), materials costing less than $1.00; and for purposes of illustration, (*g*) a pair of *billiard Balls*, suspended as in Fig. 146 (cost about $2.00).

Total cost, $9.00 to $10.00.

13. **Statics** (Exps. LXI. to LXII.): (*a*) Two *Spring Balances* of 10 kilos' capacity, graduated to $\frac{1}{10}$ Kilo (Fig. 160, page 338); cost about $2.00; (*b*) a set of *Safety-valve Weights* (Fig. 160) from 1 to 10 kilos (about $1.00); (*c*) a *Lever* (Fig. 162, page 341), costing perhaps 25 cts.; (*d*) a *loaded Board* (Figs. 170, page 348), costing about $1.00; and (*e*) a pair of *Triangular Supports* (*i* and *j*, Fig. 173, page 350).

Total cost, $4.00 or $5.00.

14. **Elasticity and Cohesion** (Exps. LXIII.–LXVII.). (*a*) Two *Steel Beams* (*ag*, Fig. 173) both 100 + *cm.* long, one 6 + *mm.* square, the other 6 + *mm.* × 12 + *mm.*, costing about $1.00; (*b*) an *Electric Micrometer* (Fig. 173), costing with platinum points about $20.00. Two students can use same micrometer. [The electrical connections are unnecessary. A common screw with a brass head soldered to it and graduated with a knife will answer. Fasten a small

metallic mirror to the beam and move screw until its point touches its own reflection.] (*c*) A *Torsion Apparatus* (Fig. 174, page 355), with 2 or 3 rods, costing perhaps $3.00; (*d*) a *Torsion Balance* (Fig. 176, page 358), made at a nominal cost by additions to the Torsion Pendulum (*ae*, Fig. 176) already mentioned (12 *d*); (*e*) *Young's Modulus Apparatus* (Figs. 178 (3), and 179, page 165), costing about $2.00, outside of the electric micrometer (see *b*); (*f*) a *Bobbin*, scooped out so as to fit over the hook of a spring balance (Fig. 180, page 367); (*g*) a *Fork* of wire for holding a film of water (page 369), and (*h*) a *Capillary Tube*, made of a broken mercury thermometer (Fig. 182, page 371). Be sure that the bore is nearly circular (not elliptical).

Total cost, $5.00 to $20.00.

15. **Work** (Exps. LXVIII. to LXX.): (*a*) a pine *Board* (20 × 100 *cm.*) *and Plank* (5 × 20 × 40 *cm.*, see Fig. 183, page 373) nicely planed (about $1.00); (*b*) a *Siphon* (Fig. 185, page 378), made by a rubber tube 2 *m.* long, 3 *mm.* internal diameter (not over $1.00); (*c*) a *Tackle* (Fig. 186), consisting of two smoothly running double blocks, of either iron or wood, to be had of dealers in shipping materials, for about $1.00; (*d*) a *Water Motor* apparatus (Fig. 178, page 382), costing about $30.00 for the motor ($\frac{1}{50}$ horse power), $3.00 for the water gauge, $1.00 for the stone jar, and $1.00 for the supports, in addition to the cost of the spring balances mentioned in 13 *a*. One motor enough for 4 or 5 students. A much smaller motor costing about $5.00 will answer; in this case letter balances

should be employed (see 20). (*e*) A *pasteboard Tube* 1 + *m.* long, 5 *cm.* diam. (sold for about 25 cts. by paper dealers), with corks (Fig. 192, page 390); (*f*) heavy iron *Weights* (1–10 *kilos*), costing from $1.00 to $2.00. Lead shot and a thermometer (1 *k*) will also be required in these experiments.

Total cost, $10.00 to $40.00.

16. **Magnetism** (Exps. LXXI. to LXXV.): (*a*) three *compound Magnets* (see ¶ 179, also Fig. 196, page 396), costing about $5.00 (one or two needed at one time); (*b*) a *vibration Magnet* (Fig. 204, page 412), cost nominal; (*c*) a *Surveying Compass* (Fig. 199, page 405), costing from $5.00 to $15.00; (*d*) a *long-bar Magnet* (Fig. 209, page 420), 1 *m.* long, 12 *mm.* diameter (cost about $1.00); (*e*) a *Dipping-needle* and stand (Fig. 210, page 423), costing about $2.00; (*f*) two wooden *Blocks* 1 *cm. cube*. Iron filings and photographic paper will be required for these experiments.

Total cost, $10.00 to $25.00.

17. **Magneto-Electricity** (Exps. LXXVI. to LXXVII.): (*a*) a *sliding Helix* with stops and clamps (*ad*, Fig. 209, page 420), costing about $1.00; (*b*) an *Earth Inductor* (Fig. 213, page 428, scale about $\frac{1}{20}$), containing about 100 turns of insulated copper wire (No. 18, B. W. G.) on a wooden ring 60 *cm.* in diameter, and costing from $5.00 to $10.00. Good results may also be obtained by a simple coil laid against a door or on a table, then suddenly turned over by hand. Cost of such a coil $2.00 or $3.00. (*c*) A *ballistic Galvanometer*, made by loading the needle of an astatic

galvanometer (Fig. 207, page 418, scale about $\frac{1}{10}$) with a few grams of lead at each end. Cost of the astatic galvanometer, about $15.00.

Total cost, $20.00 to $25.00.

18. **Galvanometry** (Exps. LXXVIII. to LXXXIV.): (*a*) a Single Ring (*S. R.*) *Galvanometer* (Fig. 217, page 438, see footnote page 439), cost without compass about $10.00; with compass $25.00; (*b*) Double-Ring (*D. R.*) *Galvanometer* (Fig. 225, page 448), cost about $10.00, exclusive of surveying compass (see 16 *c*): (*c*) electro-*Dynamometer* (Fig. 228, page 451, scale $\frac{1}{15}$), cost $10.00 or $15.00; (*d*) an *Ammeter* (Fig. 231, page 466), costing $5.00 or $10.00; (*e*) a *Vibration Galvanometer* (Fig. 230, page 461), which the student can himself construct at a cost of about 50 cts.; (*f*) a *Commutator* (Figs. 216, page 435, scale $\frac{1}{10}$), costing perhaps $1.00; (*g*) a *Shunt*, consisting of a piece of uninsulated German Silver wire of about 1 ohm resistance, with copper connections (Fig. 249, page 486), cost nominal: (*h*) several cheap *Keys*, 50 cts. to $2.00.

Batteries will also be needed as follows: (*i*) *Battery Materials*, that is, materials for a small Daniell cell to be set up in a small jar or tumbler (see page 463), cost about 50 cts.; (*j*) a *Daniell Battery* of 6 litre Daniell cells (Fig. 235, page 469), $6.00; (*k*) a *Bunsen Battery* (of 3 or 4 Bunsen cells, Fig. 234), $10.00; and (*l*) a *Leclanché Battery* of 1 or 2 Lechanché cells (Fig. 236), $2.00.

Total cost, $15.00 to $30.00.

19. **Electrical Resistance** (Exps. LXXXV. to XCIII.): (*a*) a *Resistance coil* (Fig. 238, page 471, scale

$\frac{1}{4}$), made to fit calorimeter 17 *a*) cost nominal; (*b*) a *Resistance box* (Fig. 242, page 476, see also R., Fig. 264, and Figs. 240, 241. Scale of Fig. 241 about $\frac{1}{5}$), cost, roughly adjusted, about $60.00; (*c*) a British Association (*B. A.*) *Bridge* (Fig. 247, page 481, scale of *cm.* in Fig.), cost of materials about $2.00; complete instrument $10.00 to $15.00; (*d*) a *differential Galvanometer*, made by using the differential connections of the astatic galvanometer already mentioned (17 *c*); if there are no differential connections, add to the two binding posts *a* and *b*, already existing, a third binding post, *c*. Connect *a* and *c* with a resistance of say 10 ohms; connect *b* and *c* with an *equal* resistance; then equal currents through *ac* and *bc* will produce no deflection. The instrument will work as a differential galvanometer in all cases where insulation between the two circuits is not required. Cost of change in connections about $1.00. It is preferable to have the galvanometer wound with a *double wire*, as stated in footnote, page 419.

Total cost, $60.00 to $75.00.

20. **Electromotive Force** (Exps. XCIV. to C.). These experiments depend chiefly upon the apparatus mentioned in 18 and 19. There is needed also (*a*) a *Thermo-Junction* (*a*, Fig. 257, page 521), cost nominal; (*b*) a *Clark Battery* (1 cell sufficient), cost of materials about $1.00; (*c*) an *Electric Motor* (Fig. 263, page 534), with a friction brake consisting of two letter balances (see Fig. 264), and (*d*) a *Revolution Counter* (Fig. 265). Cost of motor, &c., from $5.00 to $10.00.

Total cost, $7.00 to $12.00.

In addition to the list of apparatus given above, it will be found convenient to have certain supplies always on hand, together with tools and materials to repair broken apparatus. The most important items are arranged alphabetically below, with an estimate of the number or quantity required for each student.

Alcohol, ½ pint.
Augers, assorted.
Binding screws, electric.
[Blast lamp.]
[Brackets.]
Bunsen Burner.
Candles.
Cord, 10 feet.
Corks, assorted.
Cotton cloth, ⅛ yard.
Cotton waste, 1 oz..
Ether, 2 oz.
Gas ——.
Gimlets, assorted.
Glass Beakers, assorted.
" Jars.
" Mirrors, pieces.
" Plate, "
" Test-tubes, assorted.
" Thermometer, 1 extra.
" Tubing, assorted, but especially ¼ inch.
Hammers and Hatchets.
Ice (Exp. 5 and Exps. 22–36).
Ink.
Iron filings, ¼ oz.
[Iron Plate.]
Kerosene, 1 pint.
Mercury, 1 lb.
Nails, assorted.
Oil-can.
Paper, coördinate, 12 pages.
" photographic, 1 page.
Paraffine, ¼ oz.
Pins, assorted.
Resin, for bows.
Rubber couplings, 6.
" Stoppers, assorted.
" Tubing, assorted, 3 ft.
Salt, 1 lb.
Sand, 1 lb.
Saw.
Screw-drivers.
Screws, assorted.
Shot, lead.
" Zinc (or copper).
Solder and iron.
Tacks, assorted.
" double-pointed.
Tin-foil (10 sq. in.).
Vice.
Water, cold, hot, and distilled.
Wax.
Wire, Brass, fine, ¼ oz.
" Copper, assorted, 1 oz.
" German silver, assorted, ¼ oz.
" Iron, assorted, ¼ oz.
" Steel, fine, ¼ oz.
Wood, blocks to mount apparatus. boxes, boards, planks, strips, &c.

Every instrument, tool, or receptacle should bear the number of the shelf where it belongs, and in ad-

dition a label of its own by which it may be identified. It would be well to make an alphabetical list of apparatus referring to the shelf where each instrument may be found.[1] The students could then set up their own apparatus.

[1] The names of instruments printed in italics in the list will be found in the general index at the end of this book. The words under which they are indexed are those beginning with capital letters.

APPENDIX III.

EXPENSES.

THE cost of fitting up a laboratory for the purposes of elementary physical measurement does not differ essentially from that of an ordinary school-room, except that a somewhat greater space is required for the students. This is not, however, so great as is commonly supposed. The author has found no serious difficulty in the use of a room 15 feet square, for a section of six students. The room contained a table 9 feet square, arranged as shown in the plan (Appendix I.). The total expense of fitting up the room and table, including gas, plumbing, water, and a furnace pipe, together with shelves, drawers, closets, and a small "dumb-waiter" to connect with a tool-room in the basement, was about $300.00. Since the laboratory can receive three or four sections of 6 each daily, each member of a class of from 36 to 48 students can attend three exercises a week in a room of this size. The cost of permanent laboratory fittings is only a little greater than that of the ordinary desks required for a school-room, and if it is decided to introduce experimental physics at all, the necessary appropriation is generally forthcoming.

The cost of supplies in a course of physical measurement is not, as in chemistry, an important item.

An allowance of \$2.00 or \$3.00 per annum for each student has been found to cover the whole expense. A still smaller sum would suffice if care were taken to prevent unnecessary waste, especially of mercury, alcohol, and battery materials. The cost of gas and water is usually nominal. Heating is not included in the estimate above, and must be allowed for as in any other course of instruction.

The cost of a complete set of apparatus will be found, by adding up the separate sums in the list (Appendix II.), to vary from \$300.00 to \$500.00. A single copy of each instrument will generally answer for two students to work with at one time, especially when there are two or more instruments serving a given purpose, like the sextant and the spectrometer. In some experiments it is advisable for more than two students to work together; it is, however, recommended that one set of apparatus be allowed *on the average* for each pair of students present in the laboratory at a given time. Some instruments, like the clock and the barometer, may serve for a whole class of students; but it is highly desirable that each student should have his own thermometer, gauges, weights, and other comparatively inexpensive apparatus.

If we suppose a class of students to be divided into 6 sections, each having separate access to the laboratory at stated periods during the week, it is evident that a single complete set of apparatus can serve six pairs of students. A moderate laboratory fee (\$10.00 per student) is therefore sufficient to pay

for a complete set of apparatus in 4 or 5 years. Experience has shown that college students are willing to pay such a fee in addition to the regular charge for tuition.

It follows that 12 students can afford one complete set of apparatus, that 24 students can afford two sets, &c. In other words, the cost of reduplicating apparatus to such an extent that a large class of students working in pairs may be able without delay to follow a regular and connected course in physical measurement, comes within the limit of what the students themselves are willing to pay. There is, perhaps, no better demonstration of the fact that such a course is desirable from an economical point of view. The first criticism, however, which occurs to a practical man when he sees a whole class working simultaneously upon a given experiment, is that a great proportion of the apparatus lies idle. It may be well, therefore, to consider, from an economical point of view, certain methods of instruction in which the proportion of apparatus employed at a given time is relatively great.

"It is suggested" (see Harvard List of Advanced Physical Experiments, 1890, page 54) "that great duplication of apparatus is not necessary in a course of experiments such as is described in this" (the Harvard, 1890) "pamphlet. It is found that students differ much in their rate of working in a physical laboratory, and consequently, in more difficult experiments, the students can be working on different parts of the subject during the same hour." A

good instance of this would be in the case of experiments with double and single ring galvanometers, since the same principles are involved in the two instruments. Unfortunately, certain expensive instruments, like the balance and the resistance box, are in continual use for several weeks devoted to a given subject. To avoid conflict with such instruments, it is necessary that students should be working on *different subjects* at the same time.

This leads to the consideration of a system of instruction used in many of our oldest institutions, in which the experiments that a student is to perform are determined largely by the supply of apparatus. This system deserves especial attention, since it is in some cases the only one possible, and has the merit of extreme economy as far as apparatus is concerned.

The progress of each student in this system may be watched or controlled by an "indicator board." The names of the students can be arranged across the top of the board, and the names of experiments at the left. (See the Harvard List of advanced Physical Experiments, 1890, page 54.) A long peg is then placed under each student's name opposite the experiment which he is to perform. The long peg can be moved and replaced by a short peg, to show that the experiment has been performed, and that the apparatus is free for the next student. A complete row of short pegs opposite a given experiment shows therefore that all the students have performed it. A new experiment is then prepared in its place.

This *individual system* of instruction, aside from its economy, has undoubted educational merits. A good text-book and the personal attention of an intelligent assistant are together worth more than almost any system of lectures without such aid, as far as the understanding of experiments is concerned. When, however, we consider the mutual relations between experiments, the "individual system" as it is generally practised presents numerous defects. Professor Pickering, in his Physical Manipulation, Vol. II., appendix C., recommends that at the start about thirty experiments should be prepared. Among these are Measurements of Length, Temperature, Capacity, Weight, Force, Elasticity, Acceleration, and Light. It is true that, although some of these topics naturally precede others, any one can be explained without reference to the rest. A lecturer cannot, however, begin by explaining all. As a natural result, those students who, through lack of apparatus, are obliged to perform, for instance, experiments in light before any allusion to the wave theory has been made, work under a decided disadvantage. It is better to keep a student waiting for apparatus than for explanations. For this reason the lecturer must keep at least a month in advance of the majority of the students, even when all are working upon a given class of experiments. It is impossible after such an interval to recall the details of an explanation even with the aid of copious notes. Such details therefore are generally omitted from the lectures, and when the time comes are explained sepa-

rately to each student. Again, the lecturer cannot point out the just inferences to be drawn from a given experiment until every one has performed it. In the mean time, however, many of the results of observation escape from the student's memory. Facts without principles, like principles without facts, are quickly forgotten.

A system of lectures which is only one month in advance of the laboratory work has nevertheless many obvious advantages over a preparatory course of instruction, which must be taken at least one year in advance. It is a well-known fact that lectures are the most economical system of verbal instruction. The farther lectures are separated in time from the experiments to which they relate, the less can be accomplished by the lecturer, the more is thrown upon the laboratory assistant. In some institutions, courses of purely laboratory work are given. Such courses, with a limited number of students assigned to one instructor at a given time, have undoubted advantages, but are necessarily expensive. A small amount of apparatus may, it is true, be employed successively by a large number of students; but in an elementary course the cost of reduplicating apparatus is small in comparison with the cost of reduplicating instruction.

Let us now consider what happens when, as is frequently the case, 10 or 12 students without any previous preparation are assigned to a single assistant. In the first place, time will be lost in starting the men at work. It is easy, moreover, to see that the

assistant cannot, in a single exercise of two hours, devote more than 10 or 12 minutes to each student. In other words, the *quantity* of his instruction is limited. In the space of time at his command it would be impossible to cover the ground even of a half hour's lecture. This amount of time ought at least to be devoted to the explanation of each experiment. The *quality* of the instruction given in this way is apt, moreover, to be unsatisfactory. When the assistant has explained a given point separately to six or eight students, he is sometimes left with the impression that the point in question has been made sufficiently clear, and subsequent explanations are, perhaps involuntarily, either curtailed or omitted. The instruction obtained, even from inexperienced assistants, is often a great aid to the student in following a course of higher instruction; but without general lectures a course in physical measurement is necessarily incomplete. One of the advantages of having a course of lectures closely connected with the laboratory work, lies in the use of illustrative experiments performed by the lecturer. It would of course be too costly to repeat such experiments for the benefit of each student at the time when he needs them, and impracticable for the student to repeat most of them himself. Students working independently are left therefore to read about these experiments in a text-book, or to recall them as best they may from notes on some past lecture in which they seemed to have no practical bearing upon their work.

The student who has just heard a lecture upon a given experiment cannot fail to perform it with better understanding; he has at least the directions for the experiment fresh in his mind, and knows how to begin work. Experience has shown that more than half of the student's difficulties can be anticipated by a short lecture. There is, as has been already pointed out, a great economy of labor in this lecture method of instruction.

It has been found that one assistant is required to give individual instruction to about six students. If the class contains 72 students, whom we will suppose to be divided into 6 sections of 12 each, two assistants must evidently be present at one time, and at least three will be required to meet all the men, allowing 24 hours of instruction per week to each assistant. Now it is found that a single assistant can direct the work of 12 men at one time, provided that they have received their main instructions beforehand. Let us suppose that two assistants take 3 sections each, and explain once for all at the beginning of each exercise, in the clearest possible terms, the details of a single experiment which all the men are simultaneously to perform. Each assistant's hours will then be reduced from 24 to 18 per week, and the labor of individual instruction, if not lightened, will at least be freed from tiresome repetition. The same salary should therefore suffice. There is accordingly, in this system of instruction, a gain of one assistant's salary. There is, on the other hand, an increase in the amount of apparatus, for 6 whole sets will now

be required instead of one. The additional 5 sets will cost from $1500.00 to $2500.00. This seems a large sum to invest in apparatus; but it must be remembered that, unlike assistants' salaries, this sum is paid only once. Even if, in the course of 10 years, all the apparatus should have to be replaced, it will in the mean time, have been paid for almost twice over by the saving in a single assistant's salary.

It is obvious that instead of giving separate explanations to each section, a single lecture attended by all the sections will suffice. There is little danger that students may forget what is said in the lectures before they come into the laboratory, provided that intermediate lectures on the same subject do not intervene. This fact may be made use of, if it is desired, to effect a considerable saving in the cost of apparatus. Instead of giving a single series of lectures, the instructor may give two series, each being attended by half of the men belonging to each section of the class. It is undoubtedly possible to arrange two sets of experiments so that each may form a continuous course, and that at the same time a conflict of apparatus may be avoided. Half of the students present at a given time will accordingly be working on one set of experiments, for instance, determinations of weight, at the same time that the other half is performing an entirely different set of experiments, for instance, measurements of length. Three complete sets of apparatus will therefore be sufficient for a class of 72 students working, as suggested above, in

pairs. The details of this method have not been worked out because the expense of giving a double series of lectures would ordinarily amount in a few years to more than the original cost of reduplicating the apparatus. The method, however, involving only a single repetition of a given explanation, is obviously more economical than the ordinary system of instruction, in which explanations must be repeated separately to each student, and is to be considered, when, as is too often the case, it is absolutely impossible to obtain a sufficient appropriation for apparatus.

Let us next consider a class of 24 students, for whom one instructor or assistant is in any case sufficient. Such a class would naturally be divided into 4 sections of 6 each, and would, working in pairs, be fairly supplied by 3 complete sets of apparatus at a cost of from $900.00 to $1500.00. Suppose, however, that only one set of apparatus can be had. The simplest escape from this difficulty is to divide the class into 6 sections of 4 each, and to give a double set of lectures as has just been suggested. The exercises may if necessary be cut down to one hour each. They will then occupy, with six one-hour lectures, 24 hours per week; that is, the same time as would ordinarily be required for the individual instruction of 24 students. Each student will moreover receive the same total amount (6 hours) of instruction. Half of this will be in the lecture room, the other half in the laboratory. It would of course be better if more than one hour could be allowed for the laboratory exercises; but it is thought that the student can do

more in a single hour after a thorough explanation received in the lecture, than he could accomplish in two hours under the old system, allowing for waste of time in waiting for the necessary explanations.

We have seen that with a large class (of 72 students) it *pays* (through the saving in salaries) to reduplicate the apparatus, to such an extent at least that the students, working in pairs, may follow a regular course of experiments without waiting for apparatus. We have seen also that by means of a double course of lectures, the cost of apparatus may be considerably reduced ; in fact that, with a small class (24 students) no reduplication is necessary. With still smaller classes, 12 for instance, there is no need of reduplicating either the lectures or the apparatus. It appears, therefore, that considerations of expense arising from the reduplication of apparatus need not, as is commonly supposed, stand in the way of giving to any number of students a regular course of lectures and experiments.

So far we have considered only the minimum quantity of apparatus consistent with the purposes of this course. It is much easier at the present day to obtain a sufficient appropriation for instruction than for apparatus. It may be well, however, before leaving the subject of expense, to call attention to certain practical rules by which the cost of laboratory courses may be reduced to a minimum.

Let us suppose that the number of students, at first small, is doubled. There are then three ways of meeting this increase : 1st, by doubling the supply

of apparatus; for in this case, the same number of lectures and laboratory hours will probably suffice; 2d, by doubling the number of sections admitted to the laboratory; and 3d, by doubling the number of lectures or the number of separate explanations caused by assigning different kinds of instruments to different students at a given time. Of these three ways, the least expensive would naturally be selected. Now let the number of students again be doubled. Once more the least expensive item would be increased. The rules of economy lead, therefore, in the end to an *equal distribution* of expenses between the three above-named methods by which the capacity of a laboratory course may be increased.

For example, if it costs $300.00 per annum to give a single course of lectures, $75.00 per annum for the supervision of each laboratory section, and $50.00 per annum for interest and repairs on each complete set of apparatus, a class of 24 students, who are to work singly, should be divided into 4 sections (costing $300.00 per annum) and supplied with 6 sets of apparatus (at an equal cost of $300.00 per annum). The total cost ($900.00 per annum, or $37.50 per student) will be found to be less than that resulting from any other arrangement of sections.

The numbers chosen above were derived from experience, and represent approximately the cost of a course of physical measurement such as has been described in this book, leaving out of consideration the question of rent. Expenses may be reduced by allowing students to work in pairs, or by an increase

in the number of students; but the results compare favorably in any case with the sums charged for laboratory courses in colleges and other institutions where the individual system of instruction is still retained.[1]

[1] In the University of Berlin, the annual fee for an elementary laboratory course is from $20.00 to $25.00, in addition to a fee of $20.00 for general lectures.

APPENDIX IV.

INSTRUCTION.

THE advantages of a course of lectures closely related to the experiments which are to be performed in the laboratory, have been already pointed out. There will be needed about as many lectures (½ to 1 hour each) as there are exercises. If the class consists of a single section, the exercises in the laboratory should follow immediately after the lectures. It is well in any case for the assistant in charge of a section to say a few words to the students at the beginning of an exercise, especially in regard to detailed directions which they may require. It must not, however, be imagined that such directions can take the place of a general lecture.

Among the topics which would naturally be discussed in a general lecture, may be mentioned the historical development of the subject in hand, especially any incidents — like Rumford's boring the cannon — which may appeal to the imagination.

The laws and principles involved in the experiments should also be explained. The separate sections of Part III. cover ground enough for as many lectures. Various illustrative experiments will be found in well known text-books, such as those of Deschanel and Ganot. These should be shown, if

possible, to the whole class of students.[1] In hydrostatics, for instance, Pascal's vases, the hydrostatic press, and even the Cartesian diver may be shown; while Dr. Hall's experiments with a pressure gauge (Harvard List of Elementary Physical Experiments, No. 5) will be found to form a valuable addition to existing methods of explanation. In connection with experiments on the pressure of air, Mariotte's tube and an absolute air-pressure thermometer (¶ 75) would naturally be shown. It is useful to illustrate the expansion of gases by heat on a large scale. Experiments with Helmholtz' resonators, singing flames, and especially the phonograph, lend interest to the subject of sound; a word about photography and color are not out of place in the study of light; there are instructive experiments with powerful magnets, such as stopping the oscillations of light metallic bodies; the study of frictional electricity is also a natural introduction to the subject of electrodynamics.

Several experiments, mentioned in Parts I. and II., are not intended to be followed as determinations. Some of these are suitable for lecture illustrations; others can be performed (if it is thought desirable) by students outside of their regular course of measurements.[2] The experiments described in ¶ 80 and

[1] It is a good idea to have the students themselves take part in so far as practicable in such experiments. Notes should in all cases be taken.

[2] The growth of a desire on the part of students to perform experiments on their own account is a certain proof of progress in their past education and a promise of success in the future. Such a desire should be encouraged in every possible way.

¶ 82 may, for instance, be performed by students. If they are not, they should be shown in the lectures.

It is a good plan to dictate to the class or to write on the blackboard exactly what observations they are to make, and what calculations are to follow. Considerable time can be saved at a small expense by having these directions printed. The "hektograph" process was used for more than a year at Harvard College. Separate sheets were furnished to the students at each exercise, and handed in at the end of the exercise. The calculations were not made until afterward. In the mean time the instructor had an opportunity of examining the results of observation, so that evident mistakes could be pointed out to the students. It is important with *large classes* of students to preserve in this way some record of their original observations (see footnote, page 947). The student is, however, naturally anxious to know whether his results are satisfactory or not, and for this reason he should be allowed to take away with him a copy of his observations.

It is hardly necessary to allude to various processes, such as impression paper and the ordinary copying press, through which, if it is desired, the student's observations may be duplicated, whether they are made in ink or in pencil. It takes only a minute or two to copy figures by hand, when a printed form is already provided. This is perhaps on the whole the most satisfactory way. The printed forms should be cut and pierced so that they may be afterward bound together. All the observations made by

a given student will of course be collected by him; all the observations on a given experiment may be bound together by the instructor. It is thus easy to compare the results of different students, and to estimate the relative merits of each.

The use of printed forms is a great assistance to the instructor, for he knows exactly where to look for a given observation, and he can see at a glance if any of the necessary data have been omitted or misunderstood. At the same time, there is reason to fear that the students may fall into a mechanical way of making observations, without thinking what they are for. The student knows that he is expected to "fill in those blanks," and this he can generally do even if the reasons are not sufficiently obvious. The same objection applies to any system of instruction in which the student receives minute directions for an experiment; for it can make no essential difference whether these directions are dictated, copied, or otherwise distributed.

To test the point in question, the author has tried the following experiment. Printed forms were given to a large class of students at the end of each lecture for several months. One day, without previous notice, the lecture was closed a few minutes before the ordinary time, and each member of the class was requested to make out a form covering the observations necessary for the experiment which had been described. The determination was one which depended upon six or seven data, any one of which if missing would prevent the calculation of the result. Three-

fourths of the class presented essentially perfect forms for observation, and in addition to this, the majority named three or four additional data which would be useful in making exact corrections in the result.

It may be observed that the object of lectures is to make clear to the student what his observations are for, and what there is to learn from them. If there be any doubt whether this object is fulfilled, the natural test is an examination. To ask a student to plan out his observations is practically one form of examination; but it is one which as a general thing seems to the author unwise, because a single omission on the part of the student, unless pointed out to him, may ruin the value of his subsequent determinations.

Scientific men are frequently obliged to plan out the complete details of a determination; and it is thought desirable that students, when they have had a sufficient opportunity to see how such details are arranged, should be required in certain experiments to make their own plan for observations. At the same time, the scientific man never fails to compare his work, as far as he can, with that of others. It is not at all infrequent for him to find that he has omitted some important correction. The discovery of corrections by referring to the work of others does not incapacitate him for finding them himself. On the contrary, corrections suggest corrections. In the same way, a series of experiments in which the details are carefully and minutely planned should not incapacitate the student for making a similar arrange-

ment, but should, on the other hand, teach him how such a result may be obtained.

The principal objection to printed forms generally arises from indolent students, who see no escape from making the required number of observations. Though unnecessary in small classes, the reasonable use of printed forms is always desirable, and with a large class, greatly diminishes the labor of instruction.

The student should be taught to consider an experiment unfinished until the result has been calculated and handed in to his instructor. It will be found convenient to use cards for this purpose. The student writes his name and the name of the experiment on one side of the card, on the other side in large figures the numerical value of the result. When all the cards have been received, they may be attached in their proper place to a board bearing the names of the students at the left, and references to experiments across the top. It is thus easy to estimate at a glance both the quantity and the quality of the work performed in the laboratory.

It has been suggested (see § 30) that determinations of the properties of substances the composition of which is unknown to the student, furnish an excellent method of testing his work. Of course it will not do to give the same substance to all the students. Experiments in elementary physical measurement are divided into two distinct kinds. In one of these, the student knows what result he ought to obtain, and simply performs his experiment to test either his own skill, or the accuracy of the instruments which

he employs. Experiments in "calibration" belong to this class. The other kind of determination deals with quantities of unknown magnitude, and should be attempted by the student only when he has satisfied himself by previous experiments, that he is capable of obtaining accurate results. There is, perhaps, no greater satisfaction in a course of measurement than the discovery that quantities of absolutely unknown magnitude have been correctly measured. In estimating the value of a student's work, it is well to consider only determinations of this kind.

One word of caution is, however, necessary. Most of the materials given to the students are only commercially pure, and hence yield results which may differ indefinitely from those contained in ordinary tables. It will not do, accordingly, to assume that those results which agree most closely with these tabulated values are the best. The average result obtained by the most careful workers in a class is a much better standard; but here again caution must be observed in the case of measurements where errors tend always to increase or to diminish the result. On account of air-bubbles, for instance, the largest determinations of specific gravity are generally the best; judged, however, by the average of a class, the best results would in this case be greatly underrated. The instructor may be obliged in certain cases to make determinations himself. It is generally possible, by the use of finer apparatus or different methods, to obtain results sufficiently accurate to serve as a standard for the class.

The truest estimate of the value of a student's work, next to that furnished by a written examination, is perhaps obtained by personal inspection of the student's manner of working, and by an examination of his note-book. A word or two about note-books may not be out of place. The first and most important thing is for the student to keep his observations and his calculations separate (see § 33). If, as has been suggested, the observations are made on printed forms, there is no danger of confusion in this respect. The calculations may be made on the *back* of the printed forms or on separate sheets of paper. These calculations must in all cases be preserved. The sheets on which they are made should be of the same size and shape as those employed for observations, so that the data, calculations, and results may be bound together.

If the observations are made in an ordinary note-book, the student should follow Dr. Hall's suggestions, namely, that the left-hand pages should be devoted to observations, the right-hand pages to calculations, &c. It is a great mistake to use scraps of waste paper for arithmetical work. It is frequently necessary to review such work, and if the intermediate figures are wanting, a new calculation will be involved. The figuring, moreover, often enables an instructor to see at a glance just where a mistake was made.

Entries should be kept in so far as possible in chronological order. If mistakes are discovered later on, these should be corrected in pencil or ink of a *dif-*

ferent color from that used in the original records. These original records are often found after all to be accurate, and ought not in any case to be obliterated.[1] The use of erasers in a laboratory should be strictly prohibited.

A great and not unusual fault in note-books is a lack of sufficient fulness, or rather minuteness. The student writes, for instance, "Temperature before experiment, 50°;" without giving any idea *how long* before the experiment the temperature was taken; or again, "Length by vernier gauge, 4.01 *cm.*;" without stating by *what* vernier gauge. It would be a good plan, two or three times in a course, to have the students repeat some past experiment, making use of their notes to find the same materials and instruments that they previously employed, and to have them calculate the results without reference to any text-book. This furnishes the best test of the completeness of a student's notes.

There is a tendency on the part of some students to make their notes full by repeating explanations which are given in their text-books. This is not a very serious error; but it should be pointed out that note-books are intended for facts which a text-book cannot anticipate, and *too much* theory makes it difficult to find these facts. A good note-book is charac-

[1] "The tendency of the student to regard as unquestionably wrong any observation which is not what he expected it to be, and to make his observation tally with his expectation, is doubtless familiar to most teachers, and it should be one of the important objects of this experimental course to counteract this tendency." — *Harvard List of Elementary Physical Experiments*, page 7.

terized, not by fulness of language, but by fulness of detail.

The proceedings in an experiment should be concisely stated. Observations should be arranged as systematically as time will allow. The use of tabular forms, both for observations and for results (see "Examples," Exps. 6–10), will be found in some cases of great assistance. Calculations should be neatly made but not crowded. Generous spaces should be left between experiments, or different parts of a given experiment; and these should further be distinguished by prominent headings. An example of two pages from a note-book, with the criticisms of a teacher, is given below. A summary of results is a useful addition to the description of an experiment. Examples of such summaries will be found in the next section (V.) of this Appendix.

EXPERIMENT I.

DATE. *October 1, 1888.*

[Teacher's Remark.] *You should note that the 20 gram weight was missing.*

APPARATUS.

Block of wood,	No.	2 (*a*) *i.*
Vernier gauge,	No.	1 (*c*) *i.*
Balance,	No.	1 (*f*) *i.*
Iron weights, set	No.	1 (*g*) *i.*

OBSERVATIONS.

*Weight of the wooden block.** 122.8 *grams.*

Length of block. †	*Breadth of block.* †	*Thickness of block.* †
6.90 *cm.*	6.91 *cm.*	4.32 *cm.*
6.89	6.88	4.30
6.91	6.89	4.30
6.90	6.90	4.29
6.91	6.91	4.28
6.92	6.93	4.31
6.89	6.89	4.30
6.88	6.90	4.31
6.89	6.89	4.29
6.90	6.91	4.30

[Teacher's Remark.] *Gauge crooked? Repeat this measurement.*

[Teacher's Remark.] *Which measurement was in the middle of block?*

REMARKS.

* *The grams and tenths of grams were estimated by the small movablè weight belonging to the balance.*

† *The length was measured first parallel to the grain near one corner of the block, which was slightly broken, then at equal intervals across the block. The breadth was measured across the grain, beginning at the same corner. The thickness was measured three times near each side, and once nearly in the middle. The jaws of the gauge did not come quite together, and the two zeros did not come quite opposite.*

[Teacher's Remarks.] *Good.*

Extra observations: —

Barometer	75.32 *cm.*
Thermometer	22°.5 *C.*
Dew Point	40° *C.*

[Teacher's Remarks.] *Good practice; not needed yet.*

[Later in red ink added by student.]
This must have been Fahrenheit!

Always sign your name.

NAME...

EXPERIMENT I.

CALCULATIONS.

Length in cm.	*Breadth in cm.*	*Thickness in cm.*
6.90	6.91	4.32
6.89	6.88	4.30
6.91	6.89	4.30
6.90	6.90	4.29
6.91	6.91	4.28
6.92	6.93	4.31
6.89	6.89	4.30
6.88	6.90	4.31
6.89	6.89	4.29
6.90	6.91	4.30
10) 68.99	10) 69.01	10) 43.00
Average 6.899	*Average* 6.901	*Average* 4.300

```
   6.899            47.61
   6.901            4.300
   -----            -----
    6899             0000
   0000             0000
 62091            14283
41394            19044
---------        ---------
47.609999        204.72300 = volume in cu. cm.
```

Weight in grams.

```
204.7 )  122.8  ( 0.5999 = weight of 1 cu. cm. in grams.
         102 35
         ------
          20 450
          18 423
          ------
           2 0270
           1 8423
           ------
            18470
            18423
```

Explanatory Remarks *The volume of the block* (204.7 *cu. cm.*) *is found by multiplying together the average length* (6.899 *cm.*), *the average breadth* (6.901 *cm.*), *and the average thickness* (4.300 *cm.*). *Since* 204.7 *cu. cm. weigh* 122.8 *grams*, 1 *cu. cm. weighs* $\frac{1}{204.7}$ *of* 122.8 *grams, that is*, 0.5999 *grams, or* 0.600 *grams nearly.*

RESULTS.

Volume of the block	204.7 *cu. cm.*
Density of the block	0.600 *g. per cu. cm.*

NOTE. — An example of a fuller summary of results, which would make the explanatory remarks above unnecessary, will be found under Exp. I. Appendix V.

APPENDIX V.

EXAMPLES OF OBSERVATIONS AND CALCULATIONS IN EXPERIMENTS 1–100, PRESENTED IN THE FORM OF A SUMMARY OF RESULTS.

In the examples below, observations are printed in italics, and designated by capital letters; calculations are designated by small letters, and printed in ordinary type. The data are taken, in so far as possible, from results actually reported by students in the Jefferson Physical Laboratory, without any change whatever. Such data are marked with an asterisk referring to the initials of the name of the student by whom they were determined. Other results were obtained by calculation. In some of these, round numbers have been chosen with a view of simplifying the arithmetical work; but care has been taken in all cases to give results which either were or might have been obtained with the apparatus described in the course of experiments, and to represent correctly the probable error of such results.

EXPERIMENT I.

A.	*Weight of wooden block*	. . . .	122.8 *g.*
B.	*Length* " "	(mean of 10 obs.) . .	6.899 *cm.*
C.	*Breadth* "	" " . .	6.901 *cm.*
D.	*Thickness* "	" " . .	4.300 *cm.*
e.	Volume "	$(B \times C \times D) =$	204.7 *cu. cm.*
f.	Density "	$(A \div e) =$	0.600 *g. per cu. cm.*

EXPERIMENTS II.-IV.

Weights sinking Hydrometer.		*Temperatures of Water.*
*A.**	32.08 grams.	10°.7
*B.**	31.91 "	20°.
*C.**	31.58 "	29°.

d. Allowance for temperature at 20° about 0.03 *g.* for 1°.

☞ See Fig. 4, § 59.

NOTE. The hydrometer used above bore on the average about 1.48 grams more weight than that for which Fig. 4, § 59 was constructed. An allowance of about 1.48 grams must therefore be made in comparing results.

*E.** *Distance between* 2 *rings on hydrometer stem* . 18 *mm.*

*F.** *Weight required to sink upper ring* . . . 31.91 *g.*

*G.** *Weight required to sink lower ring* . . . . 31.88 *g.*

h. Sensitiveness of hydrometer

$\frac{1}{100}\,[E \div (F - G)] = 6$ *mm. per cg.*

*I.** *Weight sinking hydrometer in water at* 20°.5 *to mark on stem with* 12 *steel balls in upper pan* 7.59 *g.*

j. Weight would have been at 20° (see *d*) . . 7.605 *g.*

k. Apparent weight of balls in air $(B - j) =$ 24.305 *g.*

*L.** *Weight sinking hydrometer in water at* 20°.5 *with balls in lower pan* . . . . 10.713 *g.*

m. Weight would have been at 20* (see *d*) . 10.728 *g.*

n. Weight of balls in water $(B - m) =$ . . 21.182 *g.*

o. Weight of water displaced $(k - n = m - j) =$ 3.123 *g.*

p. Apparent specific gravity of balls

$(k \div o) = 7.78$ *g. per cu. cm.*

* S. L. B. Oct. 14, 1887.

EXPERIMENT V.

A. *Height of barometric column* . . . . . . 75.90 *cm.*

B. *Vertical height of barometric column when inclined so as to halve free space, about* . 75.7 *cm.*

c. Air pressure above mercury about $(A - B) =$ 0.2 *cm.*

d. Height of barometer corrected for air $(A + c) =$ 76.1 *cm.*

E. Internal diameter of barometer about . . . . 0.5 *cm.*

f. Temperature of the air of the room . . . . 20° C.

g. Correction for barometer with glass scale at 20° and 76 *cm.* (Table 18 *a*) = − 0.245 − 0.016 = ... − 0.26 *cm.*

h. Correction for capillarity, diam. 0.5 *cm.*; height of meniscus unknown (Table 18 *b*) + 0.15 *cm.*

i. Correction for mercurial vapor at 20° (Table 18 *c*) . . . . + 0.00 *cm.*

j. Corrected height of barometer $(d + g^* + h + i) =$ 76.0 *cm.*

K. Reading of Aneroid barometer . . . . 30.00 *inches.*

l. The same reduced to *cm.* (Table 16) . . . 76.2 *cm.*

m. Correction of Aneroid barometer $(l - j) = -$ 0.2(?) *cm.*

N. Moisture appears on cup (mean of 3 obs.) at . + 4° C.

O. Moisture disappears " " . + 6° C.

p. Dew-point $\frac{1}{2}(N + O)$ . . . . + 5° C.

Q. Dew-point indicated by hygrodeik . . . . + 50° F.

r. The same reduced to Centigrade (Table 39) + 10° C.

s. Correction for hygrodeik at + 10° C. $(p - r) = -$ 5° C

t. Density of dry air at 20° and 76 *cm.* (Table 19) . . . . . 0.001204 *g. per cu. cm.*

u. Correction for moisture, dew-point + 5° (Table 20) − 0.000004 *g. per cu. cm.*

v. Atmospheric density $(t + u^*) =$ 0.001200 *g. per cu. cm.*

W. Density of the air indicated by Barodeik . . . . 0.00118 " "

x. Correction for the Barodeik $(v - W) = +$ 0.00002 " "

* The corrections *g* and *u* being negative, are to be added algebraically, but subtracted numerically.

EXPERIMENT VI.

NOTE. Single weights are underlined in this table.

	A. Weights in left-hand pan (in grams).*	*B.* Weights in right-hand pan (in grams).*	*C.* First obd. turning-point of index.*	*D.* Second obd. turning-point of index.*	*E.* Third obd. turning-point of index.*	*f.* Mean of C and E.	*g.* Mean of D and *f*.	*h.* Differences.	*i.* Sensitiveness† to one centigram.
1.	0.00	0.00	8.0	10.1	8 1	8.05	9.1	2.5	1.3
2.	0.02	0.00	13.1	10.1	13.0	13.05	11.6		
3.	20.00	20.00	9.8	8.6	9.8	9.8	9.2	2.3	1.2
4.	20.02	20.00	13.1	10.0	13.0	13.05	11.5		
5.	50.00	50.00	13.2	6.2	13.1	13.15	9.7	2.1	1.1
6.	50.02	50.00	13.6	10.2	13.4	13.5	11.8+		
7.	100.00	100.00	12.2	7.8	12.1	12.15	10.0	1.5	0.8
8.	100.02	100.00	13.0	10.1	12.9	12.95	11.5		
9.	0.00	0.00	9.8	9.2	9.8	9.8	9.5		(See figure.‡)
10.	100.00	100.00	11.6	7.8	11.4	11.5	9.7		
11.	100.00	100.00	11.2	6.8	11.1	11.15	9.0		
12.	0.00	0.00	11.4	9.4	11.2	11.3	10.4		

j. Mean zero reading in last part of the experiment, ‡
$\frac{1}{2}(g_9 + g_{12}) = \frac{1}{2}(9.5 + 10.4) =$ . . 10.0

k. Mean reading of balance with 100 grams in each pan, $\frac{1}{2}(g_{10} + g_{11}) = \frac{1}{2}(9.7 + 9.0) =$ . . 9.4

l. Mean weight to be added to 100 *g* in left-hand pan to balance 100 g. in right-hand pan,
$(j - k) \div i_{7,8} = (10.0 - 9.4) \div 0.8 =$ 0.8 *cg.*

m. The same in grams . . . . 0.008 *g.*

n. Ratio of the balance-arms $(A_{10} + m) \div A_{10}$ or
$100.008 \div 100 =$ 1.00008

* W. B. B., Oct. 1887.

† The sensitiveness of this balance is not so great as that represented in Fig. 16, page 32.

‡ The variations in the zero-reading were unusually large.

EXPERIMENT VII.

EXAMPLE.

OBSERVATIONS. CALCULATIONS.

Large Weights in left-hand pan.	*Large Weights in right-hand pan.*	*Small weights, + in right hand, — in left hand pan.*	Equivalents of Weights in terms of x*.	Substitution of the value of x (1.00080 g).*	Value of Weights in grams.
A = 1-gram weight	$= x$ *	— 1 mgr.	$= x$ — 1 mgr.	= 1.0008 — .001	= 1.000
B = 2- " (No. 1).	$= x + A$	— 1 mgr.	$= 2x$ — 2 mgr.	= 2.0016 — .002	= 2.000
C = 2- " (No. 2).	$= B$	+ 2 mgr.	$= 2x$ + 0 mgr.	= 2.0016 + .000	= 2.002
D = 5- "	$= A + B + C$	+ 1 mgr.	$= 5x$ — 2 mgr.	= 5.0040 — .002	= 5.002
E = 10- " (No. 1).	$= A + B + C + D$	— 1 mgr.	$= 10x$ — 6 mgr.	= 10.0080 — .006	= 10.002
F = 10- " (No. 2).	$= E$	— 2 mgr.	$= 10x$ — 8 mgr.	= 10.0080 — .008	= 10.000
G = 20- "	$= E + F$	+ 0 mgr.	$= 20x$ — 14 mgr.	= 20.0160 — .014	= 20.002
H = 50- "	$= A + B + C + D + E + F + G$	— 3 mgr.	$= 50x$ — 36 mgr.	= 50.0400 — .036	= 50.004
I = 100- "	$= A + B + C + D + E + F + G + H$	— 4 mgr.	$= 100x$ — 73 mgr.	= 100.0800 — .073	= 100.007
J = 100- "	= 100 grams (standard)	+ 7 mgr.	$= 100x$ — 73 mgr.	" "	" "

* Since $100x$ — 73 *mgr.* (see J) = 400 *grams* + 7 *mgr.*,
$100x$ = 100 *grams* + 73 *mgr.* + 7 *mgr.* = 100.080 *grams.*
Hence x = (Sum of centigram weights) = 1.00080 *grams.*

EXPERIMENTS VIII.-X.

OBSERVATIONS.			CALCULATIONS.
A. Contents of left-hand pan.	*B. Contents of right-hand pan.*	*C. Mean indication of pointer.*	*D.* Corrected weight. §
1.* *Glass ball* [*ring No.* 1.]	102.93 *g.* [*ring No.* 2.]	10.0	102.930 *g.*
2.* 102.93 *g.* *ring No.* 1.	*Glass ball, ring No.* 2.	9.6	102.934 *g.*
3.* *Glass ball with wire in water.* †	61.82 *g.*	9.8	61.818 *g.*
4.* 61.82 *g.*	*Glass ball with wire in water.* †	11.0	61.810 *g.*
5.* 0.19 *g.*	*Wire in water.* †	10.2	0.188 *g.*
6. *Glass ball with wire in alcohol.* ‡	69.00	11.0	69.010 *g.*
7. 69.00.	*Glass ball with wire in alcohol.* ‡	10.0	69.000 *g.*
8. 0.19 *g.*	*Wire in alcohol.* ‡	10.0	0.190 *g.*

† *E.* Temperature of the water.* 18°.4.
‡ *F. Temperature of the alcohol.* 20°.0.
G. Indication of the Barodeik. .00120

§ In estimating the exact weight which would bring the pointer to No. 10 of the scale, an allowance was made at the rate of 10 mgr. for each whole division through which the pointer was deflected. This allowance corresponds to the mean sensitiveness of the balance determined in Exp. VI.

* G. H. C., Oct. 1887.

CALCULATIONS CONTINUED.

a. Anparent weight of ball in air

$$\tfrac{1}{2}(D_1 + D_2) = 102.932 \textit{ grams.}$$

b. Apparent weight of ball with wire in water $\tfrac{1}{2}(D_3 + D_4) = 61.814$ "

c. Apparent weight of wire in water $(D_5) = 0.188$ "

d. Apparent weight of ball alone in water
$(b - c) = 61.626$ *grams.*

e. Apparent weight of water displaced
$(a - d) = 41.306$ "

f. Apparent specific volume of water (Table 22) at 18°.4 (see E) in air of density .00120 (see G) = ... 1.00247 *cu. cm. per g.*

g. Volume of the ball at 18°.4 $(e \times f) =$ 41.408 *cu. cm.*

h. Weight of air displaced by ball
$(g \times G) =$ 0.050 *grams.*

i. Weight of air (Table 20, A) of the density .00120 (see G) displaced by 1 gram of brass 0.000143 "

j. Weight of air displaced by brass weights
$(a \times i) = 0.015$ "

k. Correction for the buoyancy of air
$(h - j) = 0.035$ "

l. True weight of ball in vacuo $(a + k) = 102.967$ *grams.*
The same by Table 21, assuming density of crown glass, 2.5 (Table 10), of air, .00120 (see *G*); 1.00034×102.932 (see *a*) = 102.967 *grams.*

m. Density of the ball $(l \div g) =$ . . 2.487 *g. per cu. cm.*

n. Apparent weight of ball with wire in alcohol
$\frac{1}{2}(D_6 + D_7) =$ . . . . 69.005 *grams.*

o. Apparent weight of wire in alcohol $(D_8) =$ 0.190 "

p. Apparent weight of ball alone in alcohol
$(n - o) = 68.815$ "

q. Apparent weight of alcohol displaced
$(a - p) = 34.117$ "

r. Apparent specific gravity of the alcohol
$(q \div e) = 0.826$ *g. per cu. cm.*

s. True weight of the ball in alcohol
$(p - pi) = 68.805$ *grams.*

$t.$ $\left\{ \begin{array}{l} \text{True weight (in vacuo) of alcohol displaced } (l - s) = \ldots\ldots \quad 34.162 \\ \text{The same by Table 21, for .826 (see } r\text{) and .0012 (see G);} \\ 1.00132 \times q = 1.00132 \times 34.117 = 34.162 \end{array} \right\}$ *grams.*

$u.$ Expansion of glass ball between 18°.4 (see E) and 20°0. (see F), assuming cubical coefficient 0.000023 (Table 10); $0.000023 \times (F - E) \times g = 0.000023 \times 1.6 \times 41.408 =$ 0.0015 *cu. cm.*

$v.$ Volume of glass ball at 20°.0 $(g + u) =$ 41.410 "

$w.$ Density of the alcohol at 20°.0 $(t \div v) =$ 0.8250 *g. per cu. cm.*

EXPERIMENTS XI.–XIV.

OBSERVATIONS.

A. Density of air by Barodeik 0.00120

*B. * Weight of Specific Gravity Bottle with air* 118.37 *grams.*

*C. * Weight of Sp. Gr. Bottle filled with water* 178.76 "

*D. * Temperature of the water* . . . . 22°.0 C.

---

*E. * Weight of Sp. Gr. Bottle partly filled with sand* . . . . 198.10 *grams.*

*F. * The same with spaces filled with water* . 225.29 "

*G. * Temperature of the water* . . . . 23°.1 C.

H. Weight of Sp. Gr. Bottle partly filled with sulphate of copper . . . . 185.84 *grams.*

I. The same with spaces filled with alcohol . 211.09 "

J. Temperature of the alcohol . . . . . 18°.00 C.

K. Weight of Sp. Gr. Bottle filled with alcohol (only) . . . . 168.31 *grams.*

L. Temperature of the alcohol . . . . 20°.0 C.

CALCULATIONS.

a. Apparent weight of water filling Sp. Gr. Bottle
$(C - B) =$ . . . . 60.39 *grams*

b. Apparent specific volume of water by Table 22
at 22° (see D) and .00120 (see A) . 1.00322 *cu. cm.*

c. Capacity of Sp. Gr. Bottle at 22°
$(b \times a) =$ 60.58 *cu. cm.*

---

d. Apparent weight of sand $(E - B) =$ . . 79.73 *grams.*

e. Weight of sand in vacuo by Table 21,
assuming densities 2.2 (Oxide of Silicon,
Table 9 a), and .00120 (see A),
$1.00041 \times d = 1.00041 \times 79.73 = 79.76$ "

f. Apparent weight of water filling spaces
$(F - E) = 27.19$ "

g. Apparent specific volume of water by Table 22
at 23°.1 (see G), and .00120 (see A) . . 1.00346

h. Volume of the water filling spaces
$(f \times g) = 27.28$ *cu. cm.*

i. Volume of the sand $(c - h) =$ . . . . 33.30 " "

j. Density of the sand $(e \div i) =$ . . 2.395 *g. per cu. cm.*

---

* F. W. B., Oct. 1887.

k. Apparent specific gravity of alcohol from Experiment 10 (see Examples 8, 9, and 10, *r*)
0.826 *g. per cu. cm.*

l. Apparent specific volume $(1 \div k) =$ 1.211 " "

m. Apparent weight of alcohol filling spaces
$(I - H) = 25.25$ *grams.*

n. Volume of this alcohol $(l \times m) =$ . . 30.58 *cu. cm.*

o. Volume of sulphate of copper $(c - n) =$ 30.00 " "

p. Apparent weight of sulphate of copper
$(H - B) = 67.47$ *grams.*

q. The same reduced to vacuo (Table 21), assuming densities 2.3 (Table 9 *a*) and .00120 (see *A*), $1.00039 \times p = 1.00039 \times 67.47 =$ 67.50 *grams.*

r. Density of the sulphate of copper $(q \div o) = 2.25$ *g. per cu. cm.*

---

s. Apparent weight of alcohol filling Sp. Gr. Bottle $(K - B) =$ . . . . 49.94 *grams.*

t. Weight of air (Table 20 A) of the density .00120 (see *A*) displaced by 1 *g*. of brass . 0.000143 "

u. Effective weight of the alcohol $(s - st) =$ 49.93 "

v. Weight of air filling Sp. Gr. Bottle $(A \times c) = 0.07$ "

w. Weight of alcohol in vacuo $(u + v) =$. 50.00 "

x. Difference between capacities of Sp. Gr. Bottle at 22° (see *D*) and 20° (see *L*), assuming cubical coefficient of expansion .000023 (Table 10), $0.000023 \times (D - L) \times c =$ $0.000023 \times 2 \times 60.58 = 0.003$ *cu. cm.*

y. Capacity of the Sp. Gr. Bottle at 20° $(c. - x) = 60\ 58$ "

z. Density of alcohol at 20° $(w \div y) = 0.8254$ *g. per cu. cm.*
Compare value of *x*, Examples 8–10 = 0.8250

NOTE. The strength of the alcohol corresponding to these densities (see Table 27) varies from 87.2 to 87.4 %.

EXPERIMENT XV.

OBSERVATIONS.

1. Description of Liquids.	2. Temperatures.	3. Readings of densimeter.
A. Distilled water.	21°	1.000
B. Alcohol of Exps. 8–14.	18°	0.831
C. Glycerine (commercial).	24°	1.250
D. * *Methyl alcohol.*	21°	0.814
E. * *Saturated salt solution.*	21°	1.204
F. * *Solution of bichromate of sodium.*	20°	1.470

* C. C. B., 1887.

CORRECTIONS.

a. True density of distilled water at 21° (Table 25) 0.99807

b. Density of 87.3 % alcohol (See Examples 11–14 NOTE) at 18° (see *B* 2), by Table 27 0.8269

c. Density of commercial glycerine at 24° (see C 2) according to Table 26 . . . . 1.254

d. Correction for densimeter in water $(a - A) =$ -0.002

e. Correction for " in alcohol $(b - B) =$ -0.004

f. Correction for " in glycerine $(c - C) =$ $+0.004$

☞ See Fig. 21, page 72.

g. Correction for reading in methyl alcohol † . . -0.004

h. Correction for reading in salt solution † . . . $+0.003$

i. Correction for reading in bichromate solution † about $+0.013$ (?)

j. Corrected density of methyl alcohol at 21° . . 0.810

k. Corrected density of salt solution at 21° . . . 1.207

l. Corrected density of bichromate solution at 20° . 1.49(?)

† Obtained by the curve on page 62; see § 59.

EXPERIMENT XVI.

FIRST METHOD.

A. Height of mercurial column	5.00 *cm.*
B. Height of the column of water	68.10 "
C. Temperature of the air	20° C.
d. Difference between the lengths of the columns of water and mercury $(B-A)=$ . . .	63.10 *cm.*
e. Density of air (Tables 19, 25) about . . .	.0012
f. Equivalent of inequality of air pressure in centimetres of water $(d \times e)=$	0.08 *cm.*
g. Corrected length of the column of water $(B-f)=$	68.02 *cm.*
h. Specific gravity of mercury at 20° $(g \div A)=$	13.60
i. Density of water at 20° (Table 25)	0.99828
j. Density of mercury at 20° $(h \times i)$	13.58

SECOND METHOD.

K. Height of column of glycerine	80.0 *cm.*
L. Height of column of water	100.0 *cm.*
M. Temperature of the air	20° C.
n. Difference in length of columns $(L-K)=$	20.0 *cm.*
o. Inequality of air pressure in *cm.* of water $(n \times e)=$	0.0 *cm.*
p. Corrected length of column of water $(L-o)=$	100.0 *cm.*
q. Specific gravity of glycerine at 20° $(p \div K)=$	1.250
r. Density of water at 20° (Table 25)	0.99828
s. Density of glycerine at 20° $(q \times r)=$ . . .	1.248

EXPERIMENTS XVII.–XVIII.

A.	*Weight of Flask with coal-gas*	mean of	200.500 *g.*
B.	*Weight of Flask with air*	5 double	201.200 *g.*
C.	*Weight of Flask after exhaustion*	weighings.	200.600 *g.*
D.	*Weight of Flask after admitting water* . .		700.0 *g.*
E.	*Weight of Flask completely filled with water*		1200.0 *g.*
F.	*Temperature of the water*		20° C.
G.	*Barometric pressure*		75.0 *cm.*

h. Apparent weight of water required to fill flask $(E - B) =$ . . . . 1000.0 *grams.*

i. Apparent weight of water equivalent in bulk to the air exhausted $(D - B) =$ 500.0 "

j. Degree of exhaustion $(i \div h) =$ . . . . 50 %.

k. Weight of air exhausted $(B - C) =$. 0.600 *grams.*

l. Specific gravity of this air $(k \div i)$ . . 0.00120

M. *Density of air according to Barodeik* = 0.00120

n. Specific volume of water (Table 22) at 20° (see *F*) and .00120 (see *b*) = 1.00279 *cu. cm. per g.*

o. Capacity of the flask at 20° $(h \times n) =$ 1,000.3 *cu. cm.*

p. Difference in weight between 1,000.3 *cu. cm.* of air and of coal-gas . . $(B - A) =$ 0.700 *grams.*

q. Difference of density $(p \div o) =$ 0.000700 *g. per cu. cm.*

r. Density of the coal-gas at 20° (see *F*) and 75 *cm.* (see *G*) $= (M - q) =$ . . 0.000500 *g. per cu. cm.*

s. Factor for reducing density from 20° to 0° (Table 18 *e*) 1.0734

t. Factor for reducing density from 75 *cm.* to 76 *cm.* (Table 18 *d*) 1.0133

u. Density of coal-gas at 0° and 76 *cm.*

$r \times s \times t =$ 0.00054 *g. per cu. cm.*

EXPERIMENT XIX.

I. * *Readings of Vernier Gauge set on glass ball of Exps.* 8–10.

1.	4.300 *cm.*
2.	4.303 "
3.	4.302 "
4.	4.313 "
5.	4.313 "
6.	4.300 "
7.	4.311 "
8.	4.310 "
9.	4.315 "
10.	4.311 "

A. Average 4.3078 *cm.*

B. * *Zero-reading of gauge* 0.000 *cm.*

c. Corrected diameter $(A - B) =$ . . 4.308 "

d. Apparent weight of water displaced by the glass ball (Examples 8–10, *e*) 41.306 *grams.*

e. Apparent specific volume of water (Table 22) at 18°.4 (Examples 8–10) 1.00247 *per g. cu. cm.*

f. Volume of glass ball $(d \times e) =$. 41.408 *cu. cm.*

g. Diameter of sphere (Table 3, H) with volume equal to 41.408 *cu. cm.* (see *f*) 4.293 *cm.*

h. Reduction Factor for gauge $(g \div c) =$ 0.9965

II. * *Readings of Micrometer Gauge set on steel balls of Exps.* 2–4.

1.	7.975 *revolutions.*
2.	7.990 "
3.	7.968 "
4.	7.978 "
5.	7.980 "
6.	7.981 "
7.	7.931 "
8.	7.955 "
9.	7.935 "
10.	7.968 "

A. Average 7.9664 *revolutions.*

B. * *Zero-reading of gauge* 0.00035 *rev.*

c. Corrected number of revolutions $(A - B) =$ 7.9629 "

d. Apparent weight of water displaced by 1 steel ball ($\frac{1}{12}$ of *O* in Examples 2–4) . . . 0.2603 *grams.*

e. Apparent specific volume of water (Table 22) at 20°.5 (Examples 2–4) 1.00290 *cu. cm. per g.*

f. Average volume of steel balls $(d \times e) = 0.2610$ *cu. cm.*

g. Diameter of sphere (Table 3, H.) with volume 261.0 *cu. mm.* (see *f*) . 7.929 *mm.*

h. Pitch of the micrometer screw $(g \div c) =$ 0.9957 *mm. per rev.*

* A. E. T., Nov. 29, 1887.

EXPERIMENTS XX., XXI.

READINGS OF SPHEROMETER.

	A. * *On Plane glass, 1st surface.*	*B.* * *On Lens, 1st surface.*	*C.* * *On Lens, 2d surface.*	*D.* * *On Plane Glass, 2d surface.*
1.	1.3393 *cm.*	1.1593 *cm.*	1.1595 *cm.*	1.3395 *cm.*
2.	1.3397	1.1595	1.1596	1.3393
3.	1.3397	1.1596	1.1593	1.3394
4.	1.3391	1.1593	1.1596	1.3397
5.	1.3396	1.1597	1.1594	1.3397
6.	1.3395	1.1592	1.1594	1.3396
7.	1.3395	1.1595	1.1595	1.3395
8.	1.3396	1.1595	1.1595	1.3393
9.	1.3393	1.1594	1.1596	1.3392
10.	1.3395	1.1595	1.1595	1.3392
Averages *	1.33948 *cm.*	1.15945 *cm.*	1.15949 *cm.*	1.33944 *cm.*

OBLIQUE DISTANCES OF CENTRAL POINT:—

E. * *from* 1st *foot.*	*F.* * *from* 2d *foot.*	*G.* * *from* 3d *foot.*
1. 2.218 *cm.*	1. 2.212 *cm.*	1. 2.215 *cm.*
2. 2.22 *cm.*	2. 2.200 *cm.*	2. 2.225 *cm.*
3. 2.225 *cm.*	3. 2.210 *cm.*	3. 2.248 *cm.*

*h.** Average for plane glass $\frac{1}{2}(A + D) =$. 1.33946 *cm.*

*i.** Average for lens $\frac{1}{2}(B + C) =$ . . . . 1.15947 "

*j.** Height of spherical surface $(h - i) =$. 0.17999 *cm.*

*k.** Average distance of central point from three feet (see *E*, *F*, & *G.*) . . . . . 2.219 "

*l.** Mean radius of curvature,

$$\frac{1}{2}\frac{l^2}{j} = \frac{1}{2}\frac{2.219 \times 2.219}{0.17999} = \quad . \ . \ . \ . \ 13.68 \text{ "}$$

* C. A. B., Oct. 12 and 14, 1885.

NOTE. It has been assumed in these calculations that the pitch of the spherometer screw is 1.000 *mm.* per revolution. The determination of this pitch is identical with that of a micrometer. — *See Example* 19, II.

EXPERIMENT XXII.

OBSERVATIONS.

A. Temperature of brass rod surrounded by water 20°.1

B. Length of the rod at about 20° . . . . . 1000. *mm.*

C. Reading of the micrometer . . . . 9.121 *mm.*

D. Reading of the micrometer after the admission of steam . . . . 10.643 *mm.*

E. Reading of the barometer . . . . 30.0 *inches.*

CALCULATIONS.

f. Reading of barometer, 30.0 inches (see *E*), reduced to *cm.* (Table 16) . . . 76.2 *cm.*

g. Temperature of steam at this pressure (Table 14) 100°.07

h. Increase of temperature $(g - A) =$ . . . . 80°.0

i. Expansion of rod $(D - C) =$. . . . . . 1.522 *mm.*

j. Expansion for 1° $(i \div h) =$ . . . . . . 0.01903 *mm.*

k. Expansion for 1° and for 1 *mm.* $(j \div B) =$ 0.00001903 *mm.*

l. Mean coefficient of linear expansion of brass rod between 20° and 100° in terms of its length, at 20° . . . . $k =$ 0.0000190 +

NOTE. It is assumed in these calculations that the pitch of the micrometer screw is 1.000 *mm.* per revolution. See, however, Example 19, II., in which the pitch of a similar screw is determined.

EXPERIMENT XXIII.

OBSERVATIONS.

A. Outside diameter of the tubes (mean of 4 settings on horizontal bends) . . . . 1.00 *cm.*

B. Difference of level in water-gauge due to admission of steam to left-hand jacket . . . . . . 4.03 *cm.*

C. Distance between the bends of left-hand tube when expanded by steam . . . . 99.05 *cm.*

D. Temperature of the water in right-hand jacket (mean of 3 obs. with self-registering thermometer) 18°.2 C.

E. Difference of level in water-gauge due to admission of steam to right-hand jacket . . . . . 3.98 *cm.*

F. Distance between bends of right-hand tube when expanded by steam . . . . 98.95 *cm.*

G. Temperature of the water in left-hand jacket (mean of 3 obs. with self-registering thermometer) 20°.6 C.

H. Barometric pressure . . . . 29.3 *inches.*

CALCULATIONS.

i. Barometric pressure (*H*) reduced to *cm.* (Table 16) . . . . 74.4 *cm.*

j. Temperature of steam condensing at this pressure (*i*), see Table 16 . . . . 99°.41 C.

k. Mean temperature of cold water, $\frac{1}{2}(D+G) =$ 19°.4 C.

l. Difference of temperature $(j-k) =$ . . 80°.0 C.

m. Mean length of tubes between bends, $\frac{1}{2}(C+F) =$ . . . . 99.00 *cm.*

n. Mean length of column of hot water $(m+A) =$ . . . . 100.00 *cm.*

o. Mean difference of level in gauge, $\frac{1}{2}(B+E) =$ 4.005 *cm.*

p. Mean length of column of cold water balancing the column of hot water $(n-o) =$ . . 95.995 *cm.*

q. Relative specific volume of water at 99°.4 (see *j*) and at 19°.4 (see *k*), $(n \div p) = 100.00 \div 95.995 =$ . . . 1.0417

r. Increase of specific volume per degree $(q-1) \div l = .0417 \div 80.0 =$ . . . 0.000521

NOTE. The last result (*r*) represents the mean cubical coefficient of expansion of water between 19°.4 and 99°.4, in terms of its volume at 19°.4.

EXPERIMENT XXIV.

OBSERVATIONS.

A.*	*Weight of Specific Gravity Bottle with air*	126.565 *grams.*
B.*	*Weight of Sp. Gr. Bottle with water* .	182.657 "
C.*	*Temperature of the water*	21° C.
D.*	*Weight of Sp. Gr. Bottle with alcohol*	172.49 *grams.*
E.*	*Temperature of the alcohol in D* .	16°.7 C.
F.*	*Weight of Sp. Gr. Bottle with alcohol*	171.42 *grams.*
G.*	*Temperature of the alcohol in F* .	39°.2 C.
H.*	*Weight of Sp. Gr. Bottle with alcohol*	170.43 *grams.*
I.*	*Temperature of the alcohol in H* .	59°.2 C.
J.*	*Weight of Sp. Gr. Bottle with alcohol*	169.44 *grams.*
K.*	*Temperature of the alcohol in J* .	76°.3 C.

CALCULATIONS.

a. Apparent weight of water filling Sp. Gr. Bottle $(B - A) =$ . . . 56.092 *grams.*

b. Apparent specific volume of water (Table 22) at 21° (see *C*), assuming the (mean) density of air .00120 — 1.00300 *cu. cm. per g.*

c. Capacity of Sp. Gr. Bottle at 21° $(l \times m) =$ 56.260 *cu. cm.*

d. Coefficient of cubical expansion of glass (Table 10) — 0.000023

Capacity of Sp. Gr. Bottle.

e. at 16°.7 (see *E*) $c - cd\,(C - E) =$. 56.254 *cu. cm.*

f. at 39°.2 (see *G*) $c + cd\,(G - C) =$. 56.284 "

g. at 59°.2 (see *I*) $c + cd\,(I - C) =$ . . 56.309 "

h. at 76°.3 (see *K*) $c + cd\,(K - C) =$. 56.332 "

Apparent weights of alcohol filling Sp. Gr. Bottle.

i. at 16°.7 (see *E*), $(D - A) =$ . . . . . 45.93 *grams.*

j. at 39°.2 (see *G*), $(F - A) =$ . . . . . 44.86 "

k. at 59°.2 (see *I*), $(H - A) =$ . . . . . 43.87 "

l. at 76°.3 (see *K*), $(J - A) =$ . . . . . 42.88 "

Apparent specific volumes of alcohol.

m. at 16°.7 (see *E*), $(e \div i) =$ . . 1.2248 *cu. cm. per g.*

n. at 39°.2 (see *G*), $(f \div j) =$ . . 1.2547 " "

o. at 59°.2 (see *I*), $(g \div k) =$ . . 1.2835 " "

p. at 76°.3 (see *K*), $(h \div l) =$ . . 1.3137 " "

q. at 0°, INFERRED

$(m - (p - m) \div (K - E) \times E) = 1.200$ " "

Mean coefficient of expansion in terms of the volume at 0°.

r. from 16°.7 to 39°.2, $(n - m) \div q \div (G - E) = .00111$

s. from 39°.2 to 59°.2, $(o - n) \div q \div (I - G) = .00120$

t. from 59°.2 to 76°.3, $(p - o) \div q \div (K - I) = .00147$

* G., Feb. 18, 1886.

EXPERIMENT XXV.

*A.** *Reading of the thermometer in melting snow* — 0°.1 C.

*b.** Correction of thermometer at 0° $(-A) =$ + 0°.1 C.

*C.** *Reading of barometer (reduced to cm.)* . . 76.535 *cm.*

*d.** Corresponding temperature of steam (Table 14) 100°.19 C.

*E.** *Reading of the thermometer in steam* . . . 100°.3 C.

*f.** Correction for thermometer at 100° $(d - E) =$ . . . . — 0°.1 C.

*G.** *50-degree column reaches from* 0° *up to* . . 50°.4 C.

*h.** The same would have reached from the freezing point, — 0°.1 (see *A*) up to . . . . . . 50°.3 C.

*I.** *The same reaches from* 100° *down to* . . . 51°.3 C.

*j.** The same would have reached from the normal boiling point, 100°.1 (see *f*) down to . 51°.4 C.

k.* Middle-point of thermometer, $\frac{1}{2}(h+j)=$ 50°.85 C.

l.* Correction of thermometer at 50
(50° — k) = ... —0°.85 C.

M.* *25-degree column reaches from* 0° *up to* . . 24°.8 C.

n.* The same would have reached from the freezing-point, — 0°.1 (see *A*) up to . . . . . . 24°.7 C.

O.* *The same reaches from* 50° *down to* . . . 25°.3 C.

p.* The same would have reached from the middle-point, 50°.85 (see k) down to . . . . 26°.15 C.

Q.* *The same reaches from* 50° *up to* . . . . 74°.2 C.

r.* The same would have reached from the middle-point, 50°.85 (see k) up to . . . . . . 75°.05 C.

S.* *The same reaches from* 100° *down to* . . . 76°.2 C.

t.* The same would have reached from the normal boiling-point, 100°.1 (see f) down to . 76°.3 C.

u.* First quarter-point of thermometer,
$\frac{1}{2}(n+p)=$ . . . . 25°.42 C.

v.* Correction for the thermometer at 25°
(25 — u) = ... — 0°.42 C.

w.* Last quarter-point of thermometer,
$\frac{1}{2}(r+t)$ . . . . = 75°.67 C.

x.* Correction for thermometer at 75°
(75 — w) = ... — 0°.67 C.

NOTE MADE BY STUDENT. "Notice how far off the higher readings are. I repeated the measurements several times to assure myself there was no mistake."

* C. A. E., Feb. 18, 1886.

EXPERIMENT XXVI.

CALIBRATION OF AIR THERMOMETER.

A. Depth of mercury in thermometer.	*B. Weight of thermometer and mercury.*	c. Weight of mercury ($B - B_1$).	d. Volume of mercury (.0738 × c).
1. 0.0 *cm.*	45.0 *grams.*	0.0 *grams.*	0.00 *cu. cm.*
2. 10.8 "	50.5 "	5.5 "	0.41 "
3. 18.1 "	55.0 "	10.0 "	0.74 "
4. 22.7 "	58.2 "	13.2 "	0.97 "
5. 29.5 "	63.5 "	18.5 "	1.36 "
6. 37.9 "	70.0 "	25.0 "	1.85 "
7. 43.0 "	74.1 "	29.1 "	2.15 "

☞ See Fig. 57, page 120.

E. Reading of the air thermometer in melting snow 27.3 *cm.*
F. Reading of the air thermometer in steam . . 36.1 *cm.*
G. Reading of the air thermometer in water . . 29.1 *cm.*
H. Reading of a mercurial thermometer in the same 18°.0 C.
i. Volume corresponding to *E* by interpolation between d_4 and d_5 (§ 59) . . . . . 1.23 *cu. cm.*
j. Volume corresponding to *F* by interpolation 1.73 "
k. Volume corresponding to *G* " " 1.33 "
l. Temperature of the water (formula VIII., ¶ 74), $100° \times (k - i) \div (j - i) =$ . . . . 20°. C.
m. Absolute zero of temperature (formula IX., ¶ 74), $-100° \times i \div (j - i) =$ . . . —246° C.
n. Coefficient of expansion of air (formula X., ¶ 74), $(j - i) \div i \div 100 =$ 0041

NOTE. It is not unusual to find, as in the example, variations of calibre in a tube which would, unless corrected for, introduce errors of at least 20 % into the results. A very slight quantity of moisture (about $\frac{1}{30}$ *mgr.*) in the tube of the thermometer would account for the error (about 10 %)

in the last two results (m and n). In view of such an error, the determination of temperature (l) must be considered as a means of confirming rather than correcting the reading of the mercurial thermometer (H).

EXPERIMENT XXVII.

OBSERVATIONS.

A. *Reading of air thermometer in melting snow* 273° C.
B. *Height of mercurial column in barometer* . . 75.60 *cm.*
C. *Height of mercurial column in manometer necessary to make air thermometer read* 273° *in steam* 28.00 *cm.*
D. *The same in water* . . . . 7.00 *cm.*
E. *Reading of mercurial thermometer in the water* 20°.0 C.

CALCULATIONS.

f. Temperature of steam (Table 14) at 75.60 *cm.* (see *B*) . . . . 99°.85 C.
g. Temperature of the water (1st formula, ¶ 76), $f \times D \div C = 99°.85 \times 7.00 \div 28.00 =$ 24°.96 C.
h. Absolute zero of temperature (2d formula, ¶ 76), $-f \times B \div C =$
$-\,99°.85 \times 75.60 \div 28.00 =$ . . $-269°.6$
i. The same (accepted value) . . . . $-273°$
j. Error of the determination $(h - i) \div h =$ 1 %. +
k. Coefficient of increase of pressure of confined air (3d formula, ¶ 76),
$C \div B \div f = 28.00 \div 75.60 \div 99.85 =$ .00371
l. The same (accepted value) 00367
m. Error of the determination $(k - l) \div k =$. 1 % +
n. Correction for the mercurial thermometer at 20°
$(g - E) =$ . . . . + 5°.0 C.

NOTE. In view of the comparatively close agreement (within 2 %) of the results in *h* and in *k* with accepted values, it may be assumed that the determination of temperature in *g* is accurate within a few tenths of a degree; hence the large correction (+ 5°.0) in *n* is justified. Since, however, so large a correction is improbable, the thermometer in question should be compared with one already calibrated (Exp. 25). Such a comparison might possibly show that Réaumur's (not the Centigrade) scale was employed. This would account for the results of observation.

The data in the example are sufficiently accurate to serve as a rough check (§ 45) upon the results of calibration (Exp. 25); but not as a means of correcting such results.

EXPERIMENT XXVIII.

A. Reading of mercurial barometer . . . . . 75.2 *cm.*

B. Readings of Mercurial Manometer.	*C. Readings of Air Manometer.*	*d.* Volume of the air [Example 26 *d*].	*e.* Pressure of the air [$A \pm B$].	*f.* Product of Volume and Pressure.
−25.2 *cm.*	40.75 *cm.*	2.01 *cu. cm.*	50.0 *cm.*	100.2
−15.2 "	35.08 "	1.67 "	60.0 "	100.2
− 5.2 "	30.62 "	1.48 "	70.0 "	100.1
0.0 "	29.00 "	1.33 "	75.2 "	100.0
+ 4.8 "	27.58 "	1.25 "	80.0 "	100.0
+14.8 "	25.02 "	1.11 "	90.0 "	99.9
+24.8 "	22.87 "	1.00 "	100.0 "	100.0
+34.8 "	21.53 "	0.91 "	110.0 "	100.1
+44.8 "	20.35 "	0.83 "	120.0 "	99.6
+54.8 "	19.02 "	0.77 "	130.0 "	100.1
+64.8 "	17.77 "	0.71 "	140.0 "	99.4
+74.8 "	16.72 "	0.66 "	150.0 "	99.0

G. Temperatures of boiling ether.	*H. Readings of Air Manometer.*	i. Corresponding Pressures (see *e*).
55°.0	16.91 *cm.*	148.2 *cm.*
50°.0	19.22 "	128.5 "
45°.0	21.60 "	109.5 "
40°.0	24.53 "	92.3 "
35°.0	28.20 "	78.0 "
30°.0	39.73 "	61.8 "

☞ See Fig. 65, ¶ 79.

NOTE. This example has, for simplicity, been calculated so that the products under *f* are all nearly equal to 100. The (approximate) agreement of these products follows from Mariotte's Law (§ 79), and serves as a mutual confirmation of the data under Example 26, *A* & *B*, and under Example 28, *B* & *C*, upon which these products depend.

EXPERIMENT XXIX.

OBSERVATIONS.

A. Barometric pressure . . . . 76.0 *cm.*
B. Temperature of the warm water . . . . . . 50°.0 C.
C. Weight of flask with warm water . . . . . . 50.0 *g.*
D. The same after opening under ice-water . . . 80.0 *g.*
E. Weight of flask filled with water . . . . . . 170.0 *g.*

CALCULATIONS.

f. Volume of moist air in the flask at 50° (see *B*) and 76 *cm.* (see *A*), $(E - C)$ nearly $=$ 120.0 *cu. cm.*

g. Volume of (nearly) dry air in the flask at 0° and 76 *cm.* (see *A*), $(E - D)$, nearly = . 90.0 *cu. cm.*

h. Degree of exhaustion produced by cooling to 0° $(g \div f) =$ . . . . 75.0 %

i. Pressure of the (nearly) dry air at 0° $(h \times A) =$ 57.0*cm*.

j. The same at 50° (see *B*)
$i \times (273 + B) \div 273 = i \times 323 \div 273 =$ 67.4 *cm.*

k. Additional pressure of aqueous vapor at 50° (see *B*), $(A - j) =$ . . . . 8.6 *cm.*

l. 1 *cm.* mercury (Table 49 *b*) in megadynes per *sq. cm.* = ... 0.0133

m. { *k cm.* of mercury $(kl) =$ . . . . . 0.114 } *megadynes per sq. cm.*
Difference between the pressure of aqueous vapor at 50° and at 0° by Table 13, *C* . . . . 0.117 } *megadynes per sq. cm.*

EXPERIMENT XXX.

[Mean of two or more observations.]

A.*	*Barometric pressure*	[76.00 *cm.*]
B.*	*Paraffine melts at*	54°–58°
C.*	*Alcohol boils at*	79°.2
D.*	*Chloroform boils at*	60°.6
E.*	*Bisulphide of carbon boils at*	47°.1
F.*	*Ether boils at*	35°.3

* S. L. B., Nov. 1887.

EXPERIMENT XXXI.

A.	*Weight of empty calorimeter (inner cup only)*						100.0 *grams.*		
B.	*The same nearly filled with water . . .*						180.0	"	
c.	Weight of water in calorimeter $(B - A) =$						80.0	"	
D.	*Time required to cool*	(1) *from*	80°	*to*	70°		10	*min.*	0 *sec.*
		(2) "	70°	"	60°		12	"	0 "
		(3) "	60°	"	50°		17	"	0 "
	Total	(4) "	80°	"	50°		39	"	0 "
E.	*Weight of calorimeter with a little water .*						120.0 *grams.*		
f.	Weight of the water $(E - A) = . . .$						20.0	"	
G.	*Time required when shaken*								
	to cool	(1) *from*	80°	*to*	70°		3	*min.*	20 *sec.*
		(2) "	70°	"	60°		4	"	0 "
		(3) "	60°	"	50°		5	"	40 "
	Total	(4) "	80°	"	50°		13	"	0 "
H.	*Time required without shaking*								
	to cool	(1) *from*	80°	*to*	70°		5	*min.*	0 *sec.*
		(2) "	70°	"	60°		6	"	0 "
		(3) "	60°	"	50°		8	"	30 "
	Total	(4) "	80°	"	50°		19	"	30 "
I.	*Weight of calorimeter with turpentine . . .*						175 *grams.*		
j.	Weight of turpentine $(I - A) = . . .$						75	"	
K.	*Time required to cool*	(1) *from*	80°	*to*	70°		5	*min.*	0 *sec.*
		(2) "	70°	"	60°		6	"	0 "
		(3) "	60°	"	50°		8	"	30 "
	Total	(4) "	80°	"	50°		19	"	30 "
L.	*Temperature of the room . . . .*						25°.0 C.		
m.	Difference between the weights in *c* and in *f*						60.0 *grams.*		
n.	Corresponding difference in total time of cooling $(D_4 - G_4) = ...$						26.0 *minutes*		
o.	Total time of cooling with 20 grams $(G_4) =$						13.0	"	

p. Corresponding thermal capacity
$(m \times o \div n) =$ 30.0

q. Thermal capacity of calorimeter alone
$(p - f) =$ 10.0

r. Thermal capacity of calorimeter with
turpentine $(m \times K_4 \div n) =$ 45.0

s. Thermal capacity of turpentine alone
$(r - q) =$ 35.0

t. Specific heat of the turpentine $(s \div j) =$ . . . 0.4

	Mean temperature within calorimeter.		No. of units of heat lost in 1 minute.		The same reduced to 1° difference in temperature.	
	$u =$ above 0°.	$v =$ above L.	$w = (c+q) \times 10^3 \div D$.	$x = (f+q) \times 10^3 \div H$	$y = (w \div v)$.	$z = (x \div v)$.
(1)	75°.0	50°.0	90	60	1.80	1.20
(2)	65°.0	40°.0	75	50	1.88	1.25
(3)	55°.0	30°.0	53	35	1.77	1.17
Average	65.	40.	73	48	1.8	1.2

EXPERIMENT XXXII.

FIRST METHOD.

A. *Weight of empty calorimeter (inner cup)* . 100.0 *grams.*

B. *Temperature of air within calorimeter* . 18°.0 C.

C. Temperatures of water.	*D. Times.*
1. 40°.6 (just before pouring)	10 *m.* 0 *sec.*
2. (not stationary)	11 " 0 "
3. 37°.4	12 " 0 "
4. 37°.1	13 " 0 "
5. 36°.8	14 " 0 "

E. *Weight of calorimeter with water* . . . 180.0 *grams.*

f. Rate of cooling per minute
$(C_5 - C_3) \div (D_5 - D_3) =$ . . . $0^\circ.3$ *per min.*

g. Temperature reduced to time of pouring
$(f \times (D_3 - D_1) + C_3 =$ . . . . $38^\circ.0$ C.

h. Rise of temperature of calorimeter $(g - B) =$ $20^\circ.0$ C.

i. Fall of temperature of the water $(C_1 - g) =$ $2^\circ.6$ C.

j. Weight of water $(E - A) =$ . . . . . 80.0 *grams.*

k. Units of heat given out $(i \times j) =$ . . . 208 *units.*

l. Thermal capacity of calorimeter $(k \div h) =$ 10.4

SECOND METHOD.

M. *Temperature of cold water* . . . . . . $+ 10^\circ.0$ C.

N. *Temperature of shot in calorimeter just before substitution of cold water* . . . . . . $83^\circ 0$ C.

O. *Resulting temperature* . . . . $18^\circ.0$ C.

P. *Weight of calorimeter with water* . . . . 180.0 *grams.*

q. Weight of water $(P - A) =$ . . . . . 80.0 "

r. Rise of temperature of water $(O - M) =$ $8^\circ.0$ C.

s. Units of heat absorbed by water $(q \times r) =$ 640

t. Fall of temperature of calorimeter $(N - O) =$ $65^\circ.$ C.

u. Thermal capacity of calorimeter $(s \div t) =$. 9.9

THIRD METHOD.

V. *Volume of water displaced by thermometer when immersed to the ordinary depth* . . . . 0.9 *cu. cm.*

W. *Weight of (brass) stirrer* . . . . 2.0 *grams.*

x. Thermal capacity of inner cup (brass) and stirrer (1st footnote, page 161), $0.094 \times (A + W) =$ 9.6

y. Thermal capacity of the thermometer (2d footnote, page 161), $0.46 \times V =$ 0.4

z. Total thermal capacity $(x + y) =$ . . . 10.0

EXPERIMENT XXXIII.

Determination of the Specific Heat of Lead Shot (¶ 94, I.).

[*A*.]	[*Weight of calorimeter with packing, &c., nearly filled with lead shot*] .	[797.47 *grams*].
*B.**	*The same without lead shot*	357.44 *grams*
*C.**	*Weight of bottle with ice and water before using*	501.02 "
*D.**	*Indication of the thermometer in the ice-water*	$+$ 0°.7 C.
*E.**	*Temperature of air in calorimeter* . .	23°.5 C.
*F.**	*Temperature of shot in heater*	98°. C.
*G.**	*Temperature of mixture*	22°.7 C.
*H.**	*Weight of calorimeter with mixture* . .	846.17 *grams.*
*I.**	*Weight of bottle with ice and water after using*	452.32 "
j.	Weight of water used $(C - I) =$.	48.70 "
	The same $(H - A) =$	48.70
k.	Weight of lead shot $(A - B) =$.	440.03 "
	The same $(H - B) - (C - I) =$	440.03
*l.**	Change of temperature in water $(G - D) =$	22°.0 C.
*m.**	No. of units of heat absorbed by water $(j \times l) =$	1071$+$
*n.**	Change of temperature in lead shot $(F - G) =$	75°.3 C.
o.	Thermal capacity of shot $(m \div n) =$ . . .	14.22
*p.**	Specific heat of the lead shot $(o \div k) =$ . .	0.0323 $+$

* J. E. W., Feb., 1886.

Note. The inner cup of the calorimeter employed in this determination weighed about 44 grams, and had accordingly a thermal capacity equal to about 4 grams of water. Since its temperature fell from 23°.5 to 22°.7, that is 0°.8 C., a deduction of $0.8 \times 4 = 3 +$ units should strictly be made from the number of heat units, 1071 $+$ (see *m*), apparently given out by the shot. This would make the specific heat (in *p*) 0.0320 instead of 0.0323.

EXPERIMENT XXXIV.

FIRST METHOD.

A.*	*Temperature of the room*	24°.3 C.
B.*	*Weight of bottle with kerosene*	308.9 *grams.*
C.*	*Weight of bottle with water*	267.1 "
D.*	*Temperature of kerosene*	9°.2 C.
E.*	*Temperature of water*	63°. C.
F.*	*Temperature of mixture*	24°.3 C.
G.*	*Weight of bottle with water remaining* .	. 254.6 *grams.*
H.*	*Weight of bottle with kerosene remaining*	. 241.5 "

i. Specific heat of kerosene referred to water, calculated as in the last example,

$(C - G) \times (E - F) \div (B - H) \div (F - D) =$ 0.47 +

* F. S. D., Feb., 1886.

SECOND METHOD.

J.	*Temperature of the room*	23°.0 C.
K.	*Weight of lead shot*	300.0 *grams.*
L.	*Weight of bottle with alcohol before using* .	500.0 "
M.	*Temperature of the alcohol*	+ 1°.0 C.
N.	*Temperature of the shot*	98°. C.
O.	*Temperature of the mixture*	23°.0 C.
P.	*Weight of bottle with alcohol after using* .	450.0 *grams.*

q. Specific heat of the lead shot (see the last example) 0.0320

r. Heat units given to alcohol, $q \times K \times (N - O) =$ 720

s. Specific heat of the alcohol

$r \div (L - P) \div (O - M) =$ . . . 0.65 +

Note. For a fuller statement of the calculations, see last example. On account of the agreement of the temperature of the mixture with that of the room, no allowance for the thermal capacity of the calorimeter is to be made.

EXPERIMENT XXXV.

I. Preliminary observations.

A. 10 *grams water at* 20° *with* 10 *grams alcohol at* 20° *gives mixture at* . . 28° C.
B. The same with water at 10° C. . . . . . . 21° C.
c. Temperature of water (estimated) which would give mixture at 20°, about . . . . . 8° C.

D. Weight of glass beaker used as inner cup of calorimeter 30.00 *g.*
E. The same with (about) 50 *grams of alcohol* . 80.00 *g.*
F. Temperature of the same 20°.0 C.
G. Temperature of cold water just before pouring, risen to . . . . 8°.0 C.
H. Temperature of mixture 20°.0 C.
I. Weight of the same in calorimeter . . . . 130.00 *g.*
j. Weight of alcohol $(E-D)=$ . . 50.00 *g.*
k. Weight of water $(I-E)=$ . . . 50.00 *g.*
l. Change of temperature in water $(F-G)=$ 12°.0 C.
m. No. of units of heat given out $(k \times l)=$ . . 600
n. Latent heat of mixture per gram of alcohol $(m \div j)=$ . . . 12.0

II. *Preliminary observations.*

A. 10 *grams water at* 20° *with* 1 *gram nitrate of ammonium at* 20° *gives mixture at* 14° C.
B. The same with water at 80 C. 23° C.
c. Temperature of water (estimated) which would give mixture at 20° 27° C.

D. Weight of glass beaker used as inner cup of calorimeter 30.00 *g.*
E. The same with (about) 10 *grams of nitrate of ammonium* . . . . 40.00 *g.*
F. Temperature of the same 20°.0 C.
G. Temperature of water just before pouring, fallen to 27°.0 C.
H. Temperature of mixture 20°.0 C.
I. Weight of the same in calorimeter . . . . 140.00 *g.*
j. Weight of nitrate of ammonium $(E-D)=$ 10.00 *g.*
k. Weight of water $(I-E)=$ . . . 100.00 *g.*
l. Change of temperature in water $(G-F)=$ 7°.0 C.
m. No. of units of heat absorbed $(k \times l)=$. 700
n. Latent heat of solution per gram of nitrate of ammonium $m \div j=$ 70

EXPERIMENT XXXVI.

OBSERVATIONS.

A.	*Weight of empty calorimeter (inner cup)* .	77.00 *grams.*
B.	*Weight of cotton waste with ice*	100.00 "
C.	*Temperatures of water in calorimeter* . .	*D. Times.*
1.	41°.0 C.	20 *min.* 0 *sec.*
2.	40°.5 C.	21 " 0 "
3.	[Ice transferred to calorimeter.]	22 " 0 "
4.	13°.4 C.	23 " 0 "
5.	10°.0 C.	24 " 0 "
6.	10°.0 C.	25 " 0 "
E.	*Temperature of the room*	23° C.
F.	*Weight of cotton waste*	60.00 *grams.*
G.	*Weight of calorimeter with mixture* . .	229.20 "

CALCULATIONS.

h.	Weight of ice used $(B - F) =$	40.00 *grams.*
i.	Weight of water used $(G - A - h) =$.	112.20 "
j.	Thermal capacity of (brass) cup $(.094\, A) =$	7.2
k.	Add for thermometer and stirrer (see Examp. 3)	0.6
l.	Total thermal capacity of calorimeter $(j + k) =$	7.8
m.	Thermal capacity of calorimeter with water $(i + l) =$	120.0
n.	Temperature of water reduced to time (D_3) of mixing, $C_2 - (C_1 - C_2) =$	40°.0 C.
o.	Temperature of mixture, $C_6 = C_5 =$ . .	10°.0 C.
p.	Change of temperature of water $(n - o) =$	30°.0 C.
q.	Heat units absorbed $(m \times p) =$	3600
r.	Heat units absorbed by 1 gram of ice $(q \div h) =$	90.0
s.	Heat units absorbed in raising 1 gram of *melted* ice to the temperature of the mixture, $o =$	10.0
t.	Heat required to melt 1 gram of ice $(r - s) =$	80.0

EXPERIMENT XXXVII.

OBSERVATIONS.

*A.** *Weight of brass calorimeter (inner cup)* 76.974 *grams.*
*B.** *Weight of calorimeter, thermometer, and stirrer* . . . . 99.850 "
*C.** *The same with water* . . . . 223.670 "
*D.** *Temperatures of the water at intervals of* 1 *minute:*

Before admitting steam.		During admission of steam.		After admission of steam.	
1.	8°.4	4.	20°.	6.	27°.7
2.	8°.5	5.	27°.8	7.	27°.2
3.	8°.6			8.	27°.0
				9.	27°.0

E. [*Overflow from trap*] . . . . [*none*]
F. [*Temperature of the room*] . . . . [18° ?]
*G.** *Weight of calorimeter with water and condensed steam* . . . . 227.710 *grams.*
*H.** *Barometric pressure* . . . . 74.96 *cm.*

CALCULATIONS.

i. Temperature of steam at pressure in *H* (Table 14) . . . . 99°.6 C.
j. Rate of increase of temperature before admission of steam $\frac{1}{2}(D_3 - D_1) =$. 0°.1 *per min.*
k. Rate of cooling after admission of steam $\frac{1}{2}(D_9 - D_7) =$ . . . . 0°.1 "
l. Temperature of water calculated forward to the time of 4th observation $(D_3 + j) =$ . . . 8°.7 C.
m. The same calculated backward $(D_7 + 3\,k) =$. 27°.5 C.
n. Rise of temperature $(m - l) =$ . . . . . . 18°.8 C.
o. Thermal capacity of calorimeter (see Examp. 36, *l*) 7.67

p. Weight of water in calorimeter $(C - B) =$. 123.82
q. Total thermal capacity $(o + p) =$ 131.49
r. Units of heat given out $(n \times q) =$ 2472
s. Weight of steam condensed $(G - C) =$ 4.040 *grams.*
t. Units of heat per gram of steam $(r \div s) =$ 611.9
u. Units of heat given out by 1 gram of condensed steam in cooling to the temperature of the mixture $(i - m) =$ 72.1
v. Heat given out in the condensation of 1 gram of steam $(t - u) =$ 540

* M. B., Feb. 1886.

EXPERIMENT XXXVIII.

*A.**	*Weight of zinc filings*	1.00 *gram.*
*B.**	*Weight of glass calorimeter*	23.92 "
*C.**	*Weight of battery solution*	96.[00] "
*D.**	*Temperature of battery solution* . . .	19°.5 C.
*E.**	*Time occupied by chemical action* . . .	7 *minutes.*
*F.**	*Resulting temperature (maximum)* . . .	43°.[0] C.
*G.**	*Temperature* 1.5 *minutes later*	42°.[0] C.
h.	Correction for cooling $\frac{1}{2} E \times (F - G) \div 1.5 =$	2°.3 C.
i.	Corrected temperature due to chemical action $(F + h) =$	45°.3 C.
j.	Specific heat of battery solution †	0.60
k.	Thermal capacity of the solution $(C \times j) =$	57.6
l.	Thermal capacity of glass calorimeter $(0.19\ B) =$	4.5
m.	Thermal capacity of thermometer and stirrer	0.6
n.	Thermal capacity of 1 gram of zinc (Table 8)	0.1

† The specific heat in j was taken from Table 30, for 50 % sulphuric acid, assuming that the small quantity of bichromates present would not essentially modify the result. For methods of determining the specific heat of liquids, see Exp. 32.

o.	Total thermal capacity $(k+l+m+n) =$	62.8
p.	Rise in temperature $(i-D) =$	25°.8 C.
q.	Units of heat developed $(o+p) =$. . .	1620
R.*	*Weight of zinc oxide*	1.25 *grams*.
S.*	*Weight of battery solution*	96.[00] "
T.*	*Temperature of battery solution*	17°.[0] C.
U.*	*Temperature of mixture (maximum)* . .	21°.3 C.
v.	Total thermal capacity as in *o*	62.8
w.	Rise of temperature $(U-T) =$ . . .	4°.3 C.
x.	Units of heat developed $(v \times w) =$ . .	270
y.	Difference between the number of units of heat developed by 1 gram of zinc and by its equivalent (1.25 grams) of zinc oxide $(q-y) =$	1350

* E. L. A., March, 1888.

EXPERIMENTS XXXIX.-XL.

OBSERVATIONS WITH THE	THERMO-PILE.*	PHOTOMETER.†
A. Weight of lamp before the experiment .	169. 29 *g*.	198.[0] *g*.
B. Time of lighting the lamp	2 *h*. 44 *min*.	10 *h*. 42 *min*.
C. Fixed distance of lamp from instrument	40. *cm*.	86 *cm*.
D. Weight of candle before experiment .	38.3 *g*.	26.5 *g*.
E. Time of lighting candle	2 *h*. 44 *min*.	10 *h*. 42 *min*.
F. Mean distance of candle from instrument	30.7 *cm*.‡	72 *cm*. §
G. Time of extinguishing lamp	3 *h*. 15 *min*.	10 *h*. 58 *min*.
H. Time of extinguishing candle . . .	3 *h*. 15 *min*.	10 *h*. 58 *min*.
I. Weight of lamp in middle of experiment	165.3 *g*.	195.7 *g*.
J. Time of relighting lamp	3 *h*. 20 *min*.	11 *h*. 5 *min*.
K. Fixed distance of lamp from instrument	40 *cm*.	64.5 *cm*.
L. Weight of candle in middle of experiment	34.7 *g*.	24.1 *g*.
M. Time of relighting candle	3 *h*. 20 *min*.	11 *h*. 5 *min*.
N. Mean distance of candle from instrument	29.8 *cm*.‡	60.5 *cm*. §

* F. W. A., March, 1888. † W. B. M., March, 1888.
‡ Mean of 3 observations. § Single observation.

OBSERVATIONS WITH THE	THERMO-PILE.*	PHOTOM-ETER.†
O. Time of final extinction of lamp . .	3 *h.* 50 *min.*	11 *h.* 39 *min.*
P. Time of final extinction of candle . .	3 *h.* 50 *min.*	11 *h.* 39 *min.*
Q. Weight of lamp after the experiment .	161.5 *g.*	191.8 *g.*
R. Weight of candle after the experiment .	31.2 *g.*	20.4 *g.*
CALCULATIONS — FIRST PART OF EXPERIMENT.		
a. Rate of consumption of lamp in grams per hour ‖ $(A-I) \div (G-B) =$.	7.7	8.6
b. Rate of consumption of candle in grams per hour ‖ $(D-L) \div (H-E) =$.	7.0	9.0
c. Candle power of candle $b \div 8 =$ . .	0.88	1.13
d. Candle power of lamp $C^2 \div F^2 \times c =$	1.49	1.61
e. The same reduced to 8 *g.* per hour, $8d \div a =$	1.55	1.50
CALCULATIONS — SECOND PART OF EXPERIMENT.		
f. Rate of consumption of lamp in grams per hour $(I-Q) \div (O-J) =$ . .	7.6	6.9
g. Rate of consumption of candle in grams per hour $(L-R) \div (P-M) =$.	7.0	6.5
h. Candle-power of candle $g \div 8 =$ . .	0.88	0.81
i. Candle-power of lamp $K^2 \div N^2 \times h =$	1.59	0.92
j. The same reduced to 8 grams per hour, $8i \div f =$	1.67	1.07
k. Mean relative candle-power of kerosene and paraffine under the conditions of the experiment, $\frac{1}{2}(e+j) =$ . . .	1.6 +	1.3

‖ The time of these observations is to be expressed in hours and decimal fractions of an hour.

NOTE. If the lamp and candle are weighed while burning, the observations *B, E, J, M, O,* and *P,* should read "time of weighing" instead of "time of lighting," "relighting," or "extinguishing." The calculations are identical, except that *J* and *M* are substituted respectively for observations *G* and *H,* which are omitted.

EXPERIMENT XLI.

*A.**	*Principal focal length of lens by* ORDINARY METHOD (¶ 116, 1), *mean of* 4 *observations*	13.[00] *cm.*
*B.**	*The same by* METHOD OF PARALLAX (¶ 116, 2)	— — —
*C.**	*The same by* INDIRECT METHOD (¶ 116,3)	13.287 *cm.*
*D.**	*The same by* COLOR METHOD (¶ 116, 4)	— — —
e.	Principal focal length, mean of different methods	13.14 *cm.*

EXPERIMENT XLII.

*A.**	*Nearest distance of lamp from screen consistent with perfect image* (¶ 117, 1), *mean of* 4 *observations*		51.68 *cm.*
*b.**	Principal focal length, $\frac{1}{4}$ $A =$		12.92 *cm.*
*C.**	*Distance from lamp to lens*	Conjugate focal lengths (¶ 117, 2).	49.5 *cm.*
*D.**	*Distance from lens to screen*		17.8 *cm.*
*E.**	*Distance from lamp to lens*		17.5 *cm.*
*F.**	*Distance from lens to screen*		49.8 *cm.*
g.	Mean of smaller distances		17.65 *cm.*
h.	Mean of greater distances		49.65 *cm.*
i.	Principal focal length $g \times h \div (g + h) =$		13.02 *cm.*
*J.**	*Distance from lamp to lens*	Conjugate focal lengths (¶ 117, 3).	57.8 *cm.*
*K.**	*Distance from lens to screen*		17.0 *cm.*
*L.**	*Distance from lamp to lens*		16.3 *cm.*
*M.**	*Distance from lens to screen*		58.5 *cm.*
n.	Mean of smaller distances		16.65 *cm.*
o.	Mean of greater distances		58.15 *cm.*
p.	Principal focal length $n \times o \div (n + o) =$		12.94 *cm.*
q.	Principal focal length, mean of three methods of conjugate foci, $\frac{1}{3}(b + i + p) =$ . . .		12.96 *cm.*

* C. A. B., March, 1886.

EXPERIMENT XLIII.

FIRST PART.

A. Distance of farther focus from converging lens (mean of 10 *obs.)* . . . . 90.0 *cm.*

B. Distance of nearer focus from converging lens (mean of 10 *obs.)* . . . . 30.0 *cm.*

c. Principal focal length $(A \times B) \div (A - B)$. 45.0 *cm.*

SECOND PART.

D. Distance of farther focus from diverging lens (mean of 10 *obs.)* . . . . 90.0 *cm.*

E. Distance of nearer focus from diverging lens (mean of 10 *obs.)* . . . . 30.0 *cm.*

f. Virtual principal focal length,
$(D \times E) \div (D - E) =$ . . . . . . $-$ 45.0 *cm.*

EXPERIMENT XLIV.

A. Zero reading of sextant* . . . . $0° 4' 10''$

Readings of sextant when set on sun: —

B. Positive reading (mean of* 5 *obs.)* . $0° 35' 28''$

C. Negative reading (mean of* 5 *obs.)* . $\bar{1}° + 30' 40''$

*d.** Apparent angular diameter of the sun,
$\frac{1}{2}(B - C) = \frac{1}{2}(64' 48'') =$ . . . $32' 24''$

e. Apparent "semidiameter" (d) $=$ . . $16' 12''$

f. The same reduced to decimal fraction of a degree (Table 44 *A*) . . . . $0°.270$

The same for March (Table 44, *E*) . $0°.269$

* L. L. H., March 5th, 1886.

*G.** *Zero reading of sextant* . . . . $0° \; 4' \; 10''$

Readings of sextant set on object: —

*H.** *Positive reading (mean of 5 obs.)* . . $3° \; 22' \; 34''$

*I.** *Negative reading (mean of 5 obs.)* . . $\bar{1}° + 18' \; 24''$

*j.** Apparent angular diameter of the object,
$\frac{1}{2}(H - I) = \frac{1}{2}(7° \; 4' \; 10'') =$ . . $3° \; 32' \; 5''$

k. The same reduced to decimal fraction of a degree (Table 44 *A*) . . . . $3°.535$

l. Tangent of the angle *k*, (Table 5) . . 0.0618

*M.** *Length of the object in question* . . . . $100 \; cm.$

n. Distance of the object, calculated † $(M \div l) =$ $1618 \; cm.$

*O**. *The same by measurement (from the axis of the revolving mirror to the foot of the object)* . . . . $1623 \; cm.$

* C. A. E., March, 1886.

EXPERIMENT XLV.

*A.** *Zero reading of sextant* ‡ . . . . . $2° \; 48'.5$

*B.** *Reading corresponding to 1st angle of prism* $123° \; 26'.5$

*C.** *Reading corresponding to 2d angle of prism* $121° \; 30'.0$

*D.** *Reading corresponding to 3d angle of prism* $123° \; 34'.[0]$

e. First angle, $\frac{1}{2}(B - A) = \frac{1}{2}(120° \; 18'.0) =$ $60° \; 19'.0$

f. Second angle, $\frac{1}{2}(C - A) = \frac{1}{2}(118° \; 41'.5) =$ $59° \; 20'.8$

g. Third angle, $\frac{1}{2}(D - A) = \frac{1}{2}(120° \; 45'.5) =$ $60° \; 22'.8$

h. Sum of the three angles $(e + f + g) =$ $180° \; 2'.6$

* A. E. T., March, 1888.

† A more accurate calculation may be made by the use of logarithmic tangents (Table 5, A). We have $\log n = \log M - \log \tan k$, $= 2.0000 - \bar{2}.7908 = 3.2902$; hence $n = 1619 \; cm.$

‡ The zero-reading of the instrument here employed was made purposely large so as to extend the limit of its negative readings (see Exp. 44). For second method, see next example.

EXPERIMENT XLVI.

A. Reading of telescope of spectrometer set on direct image of slit in collimator	180° 0′. 0
Readings of telescope set on image reflected:—	
B. by 1st face of prism	120° 0′. 0
C. by 2d face of prism	240° 0′. 0
d. Angle between 1st and 2d faces (¶ 126), $\frac{1}{2}(C - B) =$	60° 0′. 0

E. Reading of telescope set on image of slit in collimator illuminated by sodium flame and refracted by the prism angle (d) placed so as to produce a minimum deviation	140° 1′. 0
F. The same with prism rotated 180°	220° 1′. 0
g. Angle of minimum deviation $(A - E) =$	40° 1′. 0
h. The same $(F - A) =$	39° 59′. 0
i. The same (mean of g and h)	40° 0′. 0

NOTE. The reading of a sextant in this determination, see ¶ 127, II., would also be 40° (not 80°). Given a prism angle 60°, and an angle of minimum deviation 40°, the index of refraction is (see ¶ 244, and Table 4).

$$\mu = \sin \tfrac{1}{2}(60° + 40°) \div \sin \tfrac{1}{2}(60°) =$$
$$\sin 50° \div \sin 30° = 0.7660 \div 0.5000 = \quad . \quad 1.5320$$

EXPERIMENT XLVII.

FIRST PART.

*A.** *Zero reading of sextant* . . . . 2° 25′

Readings of sextant set upon first set of diffracted images of a thin white flame produced by a piece of linen cloth :—

*B.** *Positive reading* . . . . 2° 48′

*C.** *Negative reading* . . . . 2° 4′

d. Angular separation of images $\frac{1}{4}$ $(B - C) =$ 0° 11′.0

e. The same in decimal fraction of a degree (Table 44 *A*), $(d \div 60) =$ . . . . . . 0°. 183

f. Distance between the threads (¶ 130, formula I.), 0.00006 ÷ sin 0°.183 = 0.00006 ÷ 0. 0032 = ... 0.019 *cm.*

g. No. of threads per *cm.* $(1 \div f) =$ . . . 53

*A. E. T., March, 1888.

SECOND PART.

*H.** *Zero reading of spectrometer (mean of 3 obs.)* 29° 24′ 50″

Readings on diffracted image of slit illuminated by sodium flame :—

*I.** *Positive angle (mean of 4 obs.)* . . . . 12° 4′ 15″

*J.** *Negative angle (mean of 4 obs.)* . . . 46° 43′ 30″

*k.** Angle of minimum deviation produced by diffraction grating, $\frac{1}{2}$ $(J - I) =$ . . . . 17° 19′ 37″. 5

l. The same reduced to decimal fraction of a degree (Table 44 *A*) . . . . 17°. 327

m. Distance between lines of grating as stated by manufacturer (1 ÷ 12960) = . 0.000077160 *inches.*

n. The same reduced to *cm.* (2.5400 *m*) = 0.00019599 *cm.*

o. Length of sodium light-waves in air (¶ 130, footnote), $2 \times (n) \times \sin \frac{1}{2} (l) =$
$2 \times 0.00019599 \times 0.1506 =$. 0.0000590 + *cm.*

p. Mean of wave-lengths, D_1 & D_2 (Table 41) . . . . 0.0000589 + *cm.*

* M. B., March, 1886.

NOTE. Angles less than 25° are generally reduced with greater accuracy by a table of logarithmic sines than by a table of natural sines having the same number of places. We have by Table 4 *a* and Table 6, log 2 + log *n* + log sin $\frac{1}{2}$ *l* $= 0.30103 + \bar{4}.29224 + \bar{1}.1779 = \bar{5}.77117$; hence $o =$ 0.00005904 *cm.* (nearly). The error in this determination (0.0000001 *cm.*) corresponds to an error of about 1′ in the observed angle of diffraction.

EXPERIMENTS XLVIII., XLIX., AND L.

OBSERVATIONS.

A.	*Distance between two adjacent points of minimum sound in* ¶ 131, *I., with a small violin* A–*fork* (*mean of* 10 *obs.*)	38.0 *cm.*
B.	*The same with a small* C–*fork*	32.0 *cm.*
*C.**	*Temperature of the air within resonance tube*	22°. 75 C.
*D.**	*Relative humidity of the air of the room* . .	25 %
*E.**	*Distance between nodal points in resonance tube* (¶ 132) *due to a large* A (?)–*fork* .	74.9 *cm.*

* J. E. W., December, 1885.

F.	*The same due to a middle* C–*fork*	68.0 *cm.*
G.	*Length of monochord responding to large* A–*fork* (¶ 133, III.)	80.0 *cm.*
H.	*Length of monochord responding to middle* C–*fork*	72.0 *cm.*

CALCULATIONS.

i.	Wave-length of sound due to small violin A–fork (2 *A*) = ...	76.0 *cm.*
j.	The same due to small C–fork (2 *B*) = . .	64.0 *cm.*
k.	Velocity of sound corresponding to atmospheric conditions in *C* and *D* (see Table 15)	34.608*cm. per sec.*
l.	Wave-length due to large A–fork (2 *E*) =	149.8 *cm.*
m.	Number of vibrations of large A–fork per second (*k* ÷ *l*) = ...	230.9
	The same according to instrument maker.	228.5
n.	Wave-length of middle C–fork (2 *F*) = .	136.0 *cm.*
o.	Number of vibrations of middle C–fork per second (*k* ÷ *n*) = ...	254.5
	The same according to instrument maker	256.0
p.	Musical interval between the small A– and C–forks (*A* ÷ *B*) = ...	1.2 (nearly).
q.	Theoretical interval (6 : 5) = ...	1.20
r.	Musical interval between the large A–fork and C–fork (*E* ÷ *F*) = . . .	1.10
s.	The same (*G* ÷ *H*) = ...	1.11
t.	The same according to instrument makers, 256 ÷ 228.5 = ...	1.12

EXPERIMENT LI.

VELOCITY OF SOUND — ECHO METHOD.

*A.**	*Distance between two parallel walls* (¶ 137, II.)	80.0 *metres.*
*B.**	*Reading of metronome adjusted to keep time with echoes (mean of* 5 *observations)*	129
*C.**	*Number of beats in* 100 *seconds corresponding to this reading*	215
*D.**	*Temperature of outside air*	6° C.
*E.**	*Relative humidity*	30 %
f.	Distance traversed by sound (2 A) = . . .	160 *metres.*
g.	Time occupied (100 ÷ C) = ...	0.465 *sec.*
h.	Velocity of sound ($f \div g$) = ...	344 *metres per sec.*
i.	Velocity tabulated for conditions D and E (Table 15)	336 *metres per sec.*

* J. E. W., December, 1885.

PENDULUM METHOD.

*J.**	*Time of pendulum (on north wall of Lawrence Hall) about*	1.00 *sec.*
*K.**	*Distance of signalling observer from pendulum, about*	10 *metres.*
*L.**	*Distance of observer with telescope (Jarvis Field)*	350 ± 20 *metres.*
*m.**	Velocity of sound ($L - K$) ÷ J = ...	340 ± 20 *m. per sec.*

* Approximate results, recalled from memory from experiments made by students in 1881–1882, before the building of the Jefferson Physical Laboratory.

EXPERIMENT LII.

A. *Number of waves traced by tuning-fork between alternate marks made by pendulum (mean of* 10 *observations)* 28.27

B. *Number of complete oscillations of the pendulum timed* 100

C. *Time occupied (mean of* 10 *observations)* . . . 50.0 *sec.*

d. Rate of pendulum (complete oscillations per second) $B \div C =$ 2.00

e. Rate of tuning-fork (complete oscillations per second) $A \times d =$ . . . . 56.5 +

EXPERIMENT LIII.

NOTE. The tuning-forks Nos. 1 and 17 are supposed to have been adjusted to an exact octave (by filing or loading one of them) before the following observations were taken.

A. *No. of 1st fork.*	*B.* *No. of 2d fork.*	*C.* *Time of 100 beats.*	*d.* No. of beats per sec.	*e.* Totals.	*f.* Pitch of 1st fork ($e + e_{17}$).
1	2	25 *sec.*	4 0	0.0	65.6
2	3	20	5.0	4.0	69.6
3	4	22.2	4.5	9.0	74.6
4	5	30	3.3	13.5	79.1
5	6	27	3.7	16.8	82.4
6	7	20.8	4.8	20.5	86.1
7	8	26.3	3.8	25.3	90.9
8	9	23.8	4.2	29.1	94.7
9	10	25	4.0	33.3	98.9
10	11	29.5	3.4	37.3	102.9
11	12	23.2	4.3	40.7	106.3
12	13	20	5 0	45.0	110.6
13	14	22.3	4.5	50.0	115.6
14	15	27.2	3.7	54.5	120.1
15	16	28	3.6	58.2	123.8
16	17	26.5	3.8	61.8	127.4
17	1	*No beats.*	*No beats.*	65.6	131.2

EXPERIMENT LIV.

A. Time occupied by Lissajous' curves in passing through* 40 *complete cycles (mean of* 10 *obs.)* 18.5 *sec.*

B. Which fork must be loaded to make the figures permanent?* . . . . The higher.

C. Number of lobes visible in the symmetrical figures* ($=n$, formulae, ¶ 142). 4

d. Sign of the correction for cycles (compare B with ¶ 142) . . . . $+$

e. Number of cycles per second ($40 \div A) = c$, formulae, ¶ 142 . . . . 2.16

f. Pitch of the lower fork (Exp. 52) $= p$ in formulae, ¶ 142 . . . . 56.5 +

*g.** Pitch of the higher fork $C \times f\,[d]\,e =$ $4 \times 56.5 + 2.16 =$ . . . . 228.2 +

* M. B., December, 1885.

EXPERIMENT LV.

A. Number of revolutions made by a toothed wheel in 100 *seconds when adjusted so as to show stationary waves upon a tuning-fork (mean of* 10 *obs.)* 473

B. Number of teeth 12

c. Pitch of the tuning-fork $\frac{1}{100} A \times B$ . . 57.0

c. The same by graphical method (Exp. 52) . 56.5 +

EXPERIMENT LVI.

A. Distance between marks made by bullet . . . 19.2 *cm.*
B. Time occupied by 250 *complete oscillations of the pendulum* . . . . 200 *sec.*
c. Time occupied by pendulum after its release in reaching the middle-point of its swing,
$\frac{1}{4} \times \frac{1}{250} \times B =$ . . . . 0.200 *sec.*
d. Square of time occupied $(c^2) =$ 0.0400
e. Ratio of distance through which bullet falls to square of time occupied $(A \div d) =$ 480
f. Acceleration of gravity $(2\ e) =$ 960

[Results with other pendulums reduced in the same way and tabulated as in ¶ 148.]

EXPERIMENT LVII.

A. Distance from lower edge of bracket to top of bullet . . . . 7.8 *cm.*
B. The same to bottom of bullet . . . . 9.8 *cm.*
c. Length of pendulum $\frac{1}{2}(A + B) =$ . . . 8.8 *cm.*
D. Time occupied by 100 *single vibrations* . . 30.0 *sec.*
e. Time of pendulum $D \div 100 =$ . . . . . . 0.300 *sec.*
f. Square of time of pendulum $(e^2) =$ . . . 0.090
g. Ratio of length of pendulum to square of time $(c \div f) =$ 97.8

[Results with other pendulums reduced in the same way and tabulated as in ¶ 149.]

EXPERIMENT LVIII.

A. Diameter of rods (*ab* and *hi*, Figs. 153 and 154)	1.00 *cm.*
B. Distance between rods adjusted to	100.00 *cm.*
C. Mean interval between coincidences with seconds clock (reduced as in ¶ 152)	125 *sec.*
d. Length of pendulum $(A + B) =$ . . .	101.00 *cm.*
e. Time of pendulum $(C \div (C - 1)) =$ . .	1.00807 *sec.*
f. Acceleration of gravity corresponding to *d* and *e* (see Table, ¶ 153)	980.9

EXPERIMENT LIX.

Preliminary Observations.

A. Length of spring without load, about	50 *cm.*
B. Length of spring with bullet, about	100 *cm.*
C. Length of spring with 30 *grams*	99 + *cm.*
D. Length of spring with 31 *grams*	100 + *cm.*
E. Length of spring with 30.6 *grams, about* .	100 *cm.*
f. Weight of bullet, about	30.6 *grams.*

Observed Times of 1000 Consecutive Oscillations.

G. — With the bullet	800 *sec.*
H. — With 30.6 *grams*	801 *sec.*
I. — With 30.5 *grams*	798 *sec.*
j. Mass of the bullet, $30.5 + 0.1 \times (G - I) \div (H - I) =$	30.57 *grams.*

EXPERIMENT LX.

A. Readings of the marker at intervals of 2 sec.		*b.* Difference in 2 sec.	*c.* Mean Velocity ($b \div 2$).	*d.* Difference in 2 sec.	*e.* Acceleration ($d \div 2$).
1.	552 *mm.*				
2.	585	+ 33	+ 16.5	8.0	4.0
3.	600	+ 15	+ 7.5	10.0	5.0
4.	595	− 5	− 2.5	7.5	3.8
5.	575	− 20	− 10.0	10.0	5.0
6.	535	− 40	− 20.0		

F. Mass of the ring . . . . 500 *grams.*

G. Outside diameter of ring . . . . 20.5 *cm.*

H. Radial thickness of ring . . . . 0.5 *cm.*

i. Outside radius of ring, $\frac{1}{2}$ *G* . . . . . 10.25 *cm.*

j. Mean radius ($i - \frac{1}{2} H$) = ... 10.00 *c.m*

k. Mean deflection of outside of ring in *cm.*
(average of $A \div 10$) = ... 57.4 *cm.*

l. Mean angle of deflection in degrees,
$k \div i \times 360° \div \pi = 57.4 \div 10.25 \times 57°.3 =$ 321°

m. Acceleration of outer surface reduced to *cm.*
(average of $e \div 10$) = 0.445 *cm. per sec. per. sec.*

n. Mean acceleration of whole mass of ring
($m \times j \div i$) = ... 0.44 "

o. Force exerted by wire on ring ($F \times n$) = 220 *dynes.*

p. Couple exerted by wire on ring ($o \times j$) =
2200 *dyne-cm.*

q. Couple exerted per degree of twist
($p \div l$) = ... (nearly) 7 $\left\{\begin{array}{l}\textit{dyne-cm.}\\ \textit{per degree.}\end{array}\right.$

NOTE. The marker is here supposed to be set opposite the zero of the scale carried by the ring when the ring is at rest. The length, diameter, and material of the wire should be noted.

EXPERIMENT LXI.

Preliminary Observations.

(See Tables, pages 339 and 340, left-hand half.)

First Method.

Readings of spring balances in kilograms corrected by first table (1), *page* 339.

A. — *First balance with one end of lever* . . .	0.46	*kilos.*
B. — *Second balance with other end of lever.* .	0.44	"
C. — *First balance with load on lever*	6.85	"
D. — *Second balance with load on lever* . . .	6.75	"
e. Weight of lever $(A+B)=$	0.90	"
f. Weight of lever with load $(C+D)=$ . .	13.60	"
g. Weight of load $(f-e)=$	12.70	"

Second, Third, and Fourth Methods.

	Second Method.	Third Method.	Fourth Method.
A. [*Corrected*] *reading of spring balance bearing one end of lever*	+ 0.45	+ 0.30	+ 0.60 *kilos.*
B. The same with load on lever .	+ 6.80	− 8.20*	+ 7.80 "
c. Effect of load on lever $(B-A)$	+ 6.35	− 8.50	+ 7.20 "
D. Distance of point where spring balance is attached from fixed point of suspension	+ 100.0	+ 75.0	+ 25.0 *cm.*
E. Distance of point where load is attached from fixed point of suspension	+ 25.0	− 25 0	+ 100.0 *cm.*
f. Weight of load $(c \times D \div E) =$	25.40	25.50	1.80 *kilos.*

* *Observed reading* . . . . 7.78 *kilos.*
Correction for graduation (First Table) . + 0.10 "
Correction for inversion (180°, Second Table) + 0.32 "
Corrected reading, numerically equal to . . 8.20 "

FIFTH METHOD — OBSERVATIONS.

A.	*Corrected reading of first spring balance with load*	10.05 *kilos.*
B.	*The same for second spring balance* . .	9.95 "
C.	*Distance of point (a) from point (c) (Fig.* 166)	100.1 *cm.*
D.	*Distance of point (b) from point (c) (Fig.* 166)	99.9 *cm.*
E.	*Vertical deflection (cd, Fig.* 166) *with load*	10.00 *cm.*
F.	*Corrected reading of first spring balance without load*	9.60 *kilos.*
G.	*The same for second spring balance* . . .	9.60 "
H.	*Distance of point (a) from point (c) (Fig.* 167)	100.0 *cm.*
I.	*Distance of point (b) from point (c) (Fig.* 167)	99.8 *cm.*
J.	*Vertical deflection without load (cd, Fig.* 167)	1.04 *cm.*

CALCULATIONS.

k.	Mean reading of spring balances with load $\frac{1}{2}(A+B)=$	10.00 *kilos.*
l.	Mean length of hypothenuse with load, $\frac{1}{2}(C+D)=$	100.0 *cm.*
m.	Weight of ring with load, $2\,E\times k\div l=$	2.00 *kilos.*
n.	Mean reading of spring balances without load, $\frac{1}{2}(F+G)=$	9.60 "
o.	Mean length of hypothenuse without load, $\frac{1}{2}(H+I)=$	99.9 *cm.*
p.	Weight of ring without load $(2\,J\times n\div o)=$	0.20 *kilos.*
q.	Weight of load $(m-p)=$	1.80 "

SIXTH METHOD, ¶ 159 (6).

A. Corrected reading of spring balance pulling a point (b) in a cord (ab, Fig. 169) *to a distance (bc) from the vertical line (ac) nearly equal to* 75 *cm.* 9.65 *kilos.*

B. The same in the opposite direction (cb, Fig. 169) 9.55 "

C. Length of cord (ab, Fig. 169) . . . . . 125.0 *cm.*

D. Horizontal distance (bb', Fig. 169) . . . . 150.0 *cm.*

e. Mean deflection ($\frac{1}{2}$ *D*) = ... 75.0 *cm.*

f. Vertical distance (*ac*, Fig. 169),

$\sqrt{C^2 - e^2} = \sqrt{15625 - 5625} = \sqrt{10{,}000} =$ 100.0 *cm.*

g. Mean force felt by balance, $\frac{1}{2}(A + B)$ . . 9.60 *kilos.*

h. Weight of load,

$g \times f \div e = 9.20 \times 100.0 \div 75.0 =$ 12.80 "

EXPERIMENT LXII.

A. Weight suspended from edge of board . . . 10.0 *kilos.*

B. Distance of point of suspension from triangular support (ab, Figs. 171 *and* 172) . . . 100.0 *cm.*

C. Distance of centre of gravity from triangular support (cb,' Fig. 172) . . . . 40.0 *cm.*

d. Weight of the plank $A \times B \div C$ . . . . 25.0 *kilos.*

NOTE. The position of the centre of gravity is best located in such a heavy plank by balancing the plank upon the triangular knife-edge without the weight. See however ¶ 160.

EXPERIMENT LXIII.

	A. Length of beam between supports in cm.	*B. Breadth of the beam in cm.*	*C. Thickness of the beam in cm.*	*D. Deflection of the beam in cm.*	*E. Weight borne by the beam in grams.*	*f.* The same in dynes ($980 \times E$).	*g.* Stiffness of the beam ($f \div D$).	*h.* $\frac{f \times A^3}{B \times D \times C^3}$	*I. Material of the beam.*
1.	100.0	1.000	1.000	0.490	4,000	3.92×10^6	8.00×10^6	8.00×10^{12}	Steel No. 1.
2.	100.0	1.000	1.000	0.244	2,000	1.96 "	8.03 "	8.03 "	
3.	50.0	1.000	1.000	0.246	16,000	15.68 "	63.7 "	7.96 "	
4.	100.0	2.000	1.000	0.240	4,000	3.92 "	16.3 "	8.15 "	Steel No. 2.
5.	100.0	1.000	2.000	0.238	16,000	15.68 "	65.9 "	8.24 "	

EXPERIMENT LXIV.

PRELIMINARY EXPERIMENTS (¶ 164).

Length of rod in cm.	*Diameter of rod in cm.*	*Forces applied in kilos.*	*Points of application (Fig. 175.)*	*Length of arm in cm.*	Couple exerted, *kilog-cm.*	Deflection produced, in *degrees.*	Stiffness, *kilog-cm. per degree.*
80	1	1	*d & e*	6	6	15° +	0.40 —
80	1	1	*a & b*	6	6	15°	0.40
80	1	1	*c & d*	6	6	15° —	0.40 +
80	1	1	*a & d*	8	8	20°	0.40
80	1	1	*a & c*	10	10	25° +	0.40 —
80	1	2	*d & e*	6	12	30°	0.40
80	1	3	*d & e*	6	18	45° —	0.40 +
40	1	2	*d & e*	6	12	15°	0.80
40	1	4	*d & e*	6	24	30°	0.80
80	2	6	*a & g*	16	96	15°	6.4
80	2	10	*a & g*	16	160	25° +	6.4 —

A. Reading of needle when empty torsion balance is made horizontal . . . . + 17°

B. The same with 1 *decigram in the left hand pan* + 317°

C. The same with 1 *decigram in right hand pan,*
77°, *i. e.* . . . . —283°
D. Length of balance beam . . . . 20.4 *cm.*
E. Length of wire subject to torsion . . . . . 100.0 *cm.*
F. Diameter of the wire . . . . 0.0300 *cm.*
g. Value of 1 gram in dynes . . . . 980 *dynes.*
h. Weight of 1 decigram in dynes $(g \div 10) =$ 98.0 "
i. Length of balance arm $(\frac{1}{2} D) =$ . . . . . 10.2 *cm.*
j. Couple exerted by decigram weight $(h \times i) =$
1000 *dyne-cm.*
k. Deflection produced by this couple, $\frac{1}{2}(B - C) =$ 300°
l. Coefficient of torsion of the wire $(j \div k) =$ 3.33 { *dyne-cm. per degree.*

EXPERIMENT LXV.

A. Length of iron wire subject to stretching* . . 6505 *cm.*
B. Diameter of the wire (mean of* 20 *obs.)*
.0664 ± .0001 *cm.*
C. Reading of micrometer without weight (mean of* 10 *obs.)* . . . . 0.7446 ± .0001 *cm.*
D. The same with weight (mean of* 10 *obs.)*
1.2324 ± .0004 *cm.*
E. Weight added* . . . . 5,000 *grams.*
f. Deflection $(D - C) =$ . . . . 0.4878 *cm.*
g. Value of 1 gram in dynes . . . . 980.4 *dynes.*
h. Weight reduced to dynes
$(E \times g) =$ . . . . 4.902×10^6 *dynes.*
i. Cross-section of the wire corresponding to diameter *B*, see Table 3 G . . . . 0.003463 *sq. cm.*
j. Stress upon the wire
$(h \div i) =$ . . . . 1.416×10^9 *dynes per sq. cm.*
k. Strain of the wire $(f \div A) =$ 000750

*l.** Young's modulus of elasticity ($j \div k$) = 1.89×10^{12}

*G., January, 1885.

NOTE. From the weight (27.00 *grams*) of 10 metres of the wire, and from the density of wrought iron (7.8, Table 9) the mean cross-section would be

$27.00 \div 7.8 \div 1000 =$ . . . . . 0.00346 *sq. cm.*

EXPERIMENT LXVI.

*A.** *Diameter of steel wire* . . . . 0.02327 *cm.*

*B.** *Place of breaking.*	*C.** *Maximum reading af spring balance.*
1. *Near balance*	8.45 *kilos.*
2. *2 inches from balance*	9.61
3. *1 inch from balance*	9.30
4. *Close to balance*	9.20
5. *Close to balance*	8.50
6. *Middle*	9.68
7. *1 inch from balance*	9.66
8. *2 inches from balance*	8.95
9. *Close to balance*	8.85
10. *Middle*	8.95
*d.** *Average*	9.115 *kilos.*

NOTE BY STUDENT. *Several bad results thrown out.*

*e.** Correction of the spring balance (for zero error and graduation) for a reading of 9 kilos — 0.240 *kilos.*

*f.** Correction for an inclination of 90° . . + 0.120 "

g. Value of 1 gram in dynes . . . . . 980.4 *dynes.*

*h.** Corrected reading of spring balance ($d + e + f$) = ... 8.995 *kilos.*

*i.** The same in dynes ($h \times g$) = . . . 8.82×10^6

*j.** Cross section of wire with diameter .02327 *cm.* (see *A*), 0.000425 *sq. cm.*

*k.** Breaking stress of the steel
$(i \div j) =$ . . . . 20.8×10^9 *dynes per sq. cm.*
* J. E. W., January, 1886.

ADDITIONAL DATA.

L. Length of wire weighed . . . . 100.0 *cm.*
M. Weight in grams . . . . 0.335 *grams.*
n. Weight of 1 *cm.* $(M \div L) =$ . . . . 0.00335 "
o. Length breaking under its own weight
$(1000\, h \div n) =$ . . . . 2.69×10^6 *cm.*

EXPERIMENT LXVII.

FIRST METHOD (¶ 169 I.).

A. Distance between vertical prongs of fork . . 2.00 *cm.*
B. Weight required to counterpoise the fork when dipping into a beaker of water . 1.000 *gram.*
C. The same with film of water (mean of 10 *obs.)* . . . . 1.300 *grams.*
D. Temperature of the water . . . . 20° C.
e. Tension of film 2 *cm.* broad $(B - C) =$ 0.300 *gram.*
f. Tension of *single* surface 1 *cm.* broad
$(e \div 4) =$ . . . . 0.075 *gram.*
g. Value of 1 gram in dynes . . . . 980.4 *dynes.*
h. Surface tension of the water at 20°
(see *D*), $(f \times g) =$ . . . . 73 + *dynes per cm.*

SECOND METHOD. — CALIBRATION OF TUBE.

I. Length of mercurial column* . . . . 30.13 *cm.*
J. Weight of mercurial column* . . . . . . 7.860 *grams.*
K. Temperature of the room, about . . . . . 20° *C.*

l. Apparent specific volume of mercury at 20° (see K), from Table 23 $B =$ . . . 0.0738

m. Volume of mercury $(J \times l) =$ . . . . . 0.580 *cu. cm.*

n. Cross-section of the tube $(m \div I) =$. 0.0192 *sq. cm.*

o. Diameter corresponding (see Table 3 *G*) 0.156 + *cm.*

HEIGHT OF CAPILLARY COLUMN.

*P.** *Height to which water rises in the tube above its level outside of the tube (mean of* 5 *obs.)* 1.66 ± .01 *cm.*

q. Density of water at 20° (Table 25) 0.99828

r. Density of air at 20° (mean), Table 19 . . 0.00120

s. Weight of 1 *cu. cm.* of water at 20° in air, $(q - r) =$ 0.99708

t. Weight in air of a column of water 1.66 *cm.* long, 0.0192 *sq. cm.* in cross-section, reduced to dynes $(P \times n \times s \times g) =$ . . . . 31.0 *dynes.*

u. Breadth of film sustaining the weight of this column = circumference of tube (Table 3, F), with diameter 0.156 *cm.* (see *D*) . . . 0.490 *cm.*

v. Surface tension of the water at 20° $(t \div u) =$ 63 + *dynes per cm.*

* A. N. S., January, 1887.

EXPERIMENT LXVIII.

FIRST METHOD (¶ 171, I.).

A. Force required to draw plank with uniform velocity parallel to the fibres of the plank and of a horizontal board upon which the plank rests flatwise (mean of 10 *obs.)* . . . . . . 0.290 *kilo.*

B. The same with plank edgewise . . . . . 0.310 "

C. The same with plank flatwise but bearing a load 3.50 *kilos.*

D. *Weight of the plank* . . . . 1.00 *kilo.*

E. *Weight of the load* . . . . 10.00 *kilos.*

f. Coef. of friction (1) in A $(A \div D) =$ . . . 0.29
" " (2) " B $(B \div D) =$ . . . 0.31
" " (3) " C $(C \div (D + E)) =$. 0.32

SECOND METHOD (¶ 171, II.).

G. *Distance* (*AB*, Fig. 184) *measured along horizontal surface of table from the point of contact of the under surface of board to foot of vertical measuring rod* . . . . 100.0 *cm.*

H. *Height of under surface of board above horizontal surface of table at this point sufficient to make plank slide down board with uniform velocity parallel to the fibres of the plank and of the board with the plank flatwise* (*mean of* 10 *obs.*) 30.0 *cm.*

i. Slope of under surface of board = slope of upper surface nearly = coefficient of friction $= H \div G =$ 0.30

EXPERIMENT LXIX.

OBSERVATIONS.

*A.** *Circumference of the wheel of motor* . . 72.8 *cm.*

	1st	2d	3d	4th trial.
*B.** [*Difference between*] *readings of spring balance* [*s*]	1.3	1.2	1.1	1.0 *kilos.*
*C.** *No. of revolutions made by the wheel*	14	15	18	18 *rev.*
*D.** *Duration of the experiment in seconds*	10	10	10	10 *sec.*
*E.** *Weight in grams of the water used* .	2190	2360	2640	2514 *grams.*
*F.** *Readings of the pressure-gauge in pounds per square inch*	20	16.5	19.5	19 *lbs. per sq. in.*

CALCULATIONS.

	*1st	2d	3d	4th *trial.*	
g. Tangential force of friction reduced to megadynes (0.98 *B*) = . . .	1.27	1.18	1.08	0.98	*mega-dynes.*
h. Velocity of rim of wheel in *cm.* per sec. ($A \times C \div D$)=	102	109	131	131	*cm. per sec.*
i. Power utilized by motor in megergs per sec. ($g \times h$) = ...	130	129	141	128	*megergs per sec.*
j. Pressure of water reduced to megadynes per *sq. cm.* ($0.069 \times F$) =	1.88	1.14	1.35	1.31	*megad. per. sq. cm.*
k. Flow of water in *cu. cm.* per sec. ($1.00 \times E \div D$) = ...	219	236	264	251	*cu. cm. per sec.*
l. Power spent on motor in megergs per second ($j \times k$) = ...	302	269	356	329	*megergs per sec.*
m. Efficiency of motor in per cent $100 \times i \div l$ =.	43	48	40		39 %

* P. M. H., January, 1886.

EXPERIMENT LXX.

OBSERVATIONS.

A. Length of pasteboard tube with corks . . 124.0 *cm.*

B. Thickness of corks each . . . . 2.0 *cm.*

C. Depth of lead shot (by difference) . . . 20.0 *cm.*

D. Temperature of the room . . . . 20°.0 C.

E. Temperature of the shot before the experiment reduced to * . . . . 17°.0 C.

*F. Temperature of the pasteboard tube before the experiment raised to** . . . . 23°.0 C.

G. Temperature of the shot and tube after the experiment . . . . 23°.0 C.

H. Number of reversals necessary to bring about this change of temperature in the shot . . 81

CALCULATIONS.

i. Rise of temperature of the shot $(G - F) =$ 6°.0 C.

j. Distance fallen by the shot in each reversal
$(A - 2B - C) =$ . . . . 100.0 *cm.*

k. Total distance fallen by the shot $(H \times j) =$ 8100 *cm.*

l. Distance fallen per degree rise of temperature
$(k \div i) =$ . . . . 1350 *cm.*

m. Force acting upon each gram of shot . . 980.4 *dynes.*

n. Work necessary to raise 1 gram of shot 1° in temperature reduced to megergs
$(l \times m \div 1000000) =$ . . . . 1.324 *megergs.*

o. Heat units necessary to raise 1 gram of lead 1° C. (See Exp. 31, and Table 8) 0.032 *units of heat.*

p. Mechanical equivalent of heat*
$(n \div o) =$ . . . . .42 $\left\{ \begin{array}{l} \textit{megergs per} \\ \textit{unit of heat.} \end{array} \right.$

* A simple way to cool shot to a given temperature is to mix it with colder shot from a refrigerator. The tube may be warmed to a given temperature by placing shot in it a little above that temperature. Assuming that, as in the example, the operations have been performed, so that $\frac{1}{2}(F + E) = D$, and $G = F$, the effects of cooling and thermal capacity will be eliminated (see ¶¶ 98, 102 and 104), and the probable error of the result due to other causes (see ¶ 178) ought not to exceed 5 %.

EXPERIMENTS LXXI.-LXXIV.

OBSERVATIONS — *Magnet numbered*	1.	2.	3.
A. Distance between the poles of the magnet in centimetres . . .	9.6	10.0	10.4 *cm.*
B. Weight in grams necessary to counterpoise each magnet	240.00	250.00	260.00 *grams.*
C. The same when repelled as follows: No. 1 by No. 2; No. 2 by No. 3; No. 3 by No. 1	239.36	249.30	259.33 "

OBSERVATIONS — *Magnet numbered*	1.	2.	3.
D. The same when attracted as stated	241.26	251.40	261.33 "
E. Distance between the magnets from centre to centre (in cm.) . . .	2.00	2.00	2.00 *cm.*
F. Zero-reading of torsion-apparatus	0°.0	0°.0	0°.0
G. Reading of the same when magnet points east and west	+ 117°.0	+ 120°.0	+ 123°.0
H. Reading of the same when magnet points west and east	− 117°.0	− 120°.0	− 123°.0
I. Mean distance of centre of magnet from centre of compass needle, measured in east and west positions	35.0	35.0	35.0 *cm.*
J. Readings of compass needle with magnet east of needle			
(*Fig.* 200,1) *North pole deflected westward [N. W.]* . .	8°.8	9°.8	10°.9
(*Fig.* 200,1) *South pole deflected eastward [S. E.]* . .	8°.6	9°.4	10°.7
(*Fig.* 200,2) *North pole deflected eastward [N. E.]* . .	8°.5	9°.5	10°.6
(*Fig.* 200,2) *South pole deflected westward [S. W.]* . .	8°.9	9°.7	11°.0
With magnet west of needle.			
(*Fig.* 200,3) *North pole deflected westward [N. W.]* . .	8°.6	9°.5	10°.8
(*Fig.* 200,3) *South pole deflected eastward [S. E.]* . .	8°.4	9°.3	10°.4
(*Fig.* 200,4) *North pole deflected eastward [N. E.]* . .	8°.3	9°.2	10°.5
(*Fig.* 200,4) *South pole deflected westward [S. W.]* . .	8°.7	9°.6	10°.7
CALCULATIONS.			
a. Mean force felt by each magnet, $\frac{1}{2}(D-C) =$	0.950	1.050	1.000 *grams.*
b. Mean force felt by each pole, $(\frac{1}{2}a) =$	0.475	0.525	0.500 "
c. The same in dynes $(980.4 \times b) =$	466	515	490 *dynes.*
d. The same reduced to a distance of 1 *cm.* giving product of strength of poles acting, $(c \times E^2) =$	1864	2060	1960 *dynes.*

CALCULATIONS — *Magnet numbered.*	1.	2.	3.
e. Ratio of the strengths of poles in question $(d_3 \div d_2)$; $(d_1 \div d_3)$; $(d_2 \div d_1)$ = respectively . .	0.951	0.951	1.105
f. Provisional estimate of strength of single poles, $\sqrt{d \times e}$ = .	42.1	44.3	46.5 { *units of magnetism.*
g. Coefficient of torsion of wire (Exp. 64) in *dyne-cm. per degree*	3.33	3.33	3.33 { *dyne-cm. per degree.*
h. Mean angle of torsion observed, $\frac{1}{2}(G-H)-90°$ = . . .	27°.0	30°.0	33°.0
i. Couple exerted by or upon magnet in *dyne-cm.* $(g \times h)$ =. .	90.0	100.0	110.0 { *dyne-cm.*
j. Force of earth's magnetism on each pole $(i \div A)$ = . . .	9.4	10.0	10.6 *dynes.*
k. 1st estimate of the horizontal intensity of earth's magnetism $(j \div f)$ = ...	0.223	0.226	0.228
l. Distance of nearer pole of magnet from centre of compass needle $(I-\frac{1}{2}A)$ =. . . .	30.2	30.0	29.8 *cm.*
m. Distance of further pole $(I+\frac{1}{2}A)$ = ...	39.8	40.0	40.2 *cm.*
n. Field of force due to nearer pole $(f \div l^2)$ =.	0.0462	0.0492	0.0524
o. Field of force due to farther pole $(f \div m^2)$ = ...	0.0266	0.0277	0.0288
p. Resultant field of force $(n-o)$ =	0.0196	0.0215	0.0236
q. Mean angle of deflection (average of J)	8°.6	9°.5	10°.7
r. Tangent of this angle (Table 5)	0.1512	0.1673	0.1890
s. 2d estimate of the horizontal intensity of the earth's magnetism $(p \div r)$ = ...	0.130	0.129	0.125
t. The same (1st estimate — see *k*)	0.223	0.226	0.228
u. Geometric mean between the two estimates = Horizontal intensity of earth's magnetism = $\sqrt{s \times t}$ =.	0.170	0.171	0.169
v. "Moment" of magnets $(i \div u)$ =	529	585	651
w. Strength of poles $(v \div A)$ =.	55	58	63

EXPERIMENT LXXV.

*A.** *Number of vibrations made by a small loaded magnetic needle in* 10 *seconds under the influence of the earth's magnetism* . . . 1.5 *vibr.*

*B.** *The same at a distance of* 1 *cm. from the middle point of a long bar magnet* . . . 8.0 *vibr.*

The same at a distance of 1 *cm. from the axis of the magnet, and at the following distances :—*

	From north end —	*Vibrations.*		*From south end —*	*Vibrations.*
*C.**	0 *cm.*	17.0	*H.**	0 *cm.*	18.0
*D.**	10 *cm.*	13.0	*I.**	10 *cm.*	14.5
*E.**	20 *cm.*	12.5	*J.**	20 *cm.*	11.0
*F.**	30 *cm.*	12.0	*K.**	30 *cm.*	7.5
*G.**	40 *cm.*	11.0	*L.**	40 *cm.*	6.0

☞ See Fig. 205, page 413, representing the squares of the numbers of vibrations.

Inferences from this Figure.

m. Distance of north pole from end of magnet . . 22 *cm.*
n. Distance of south pole from end of magnet . . 12 *cm.*
o. Distance between the poles . . . . 66 *cm.*

* C. R. E., April, 1888.

EXPERIMENT LXXVI.

A. Throws of needle of ballistic galvanometer caused by sliding a coil over the magnet through a distance of* 10 *cm., measured as follows:* —

From north end of magnet		*— Throw.*	*From south end of magnet*		*— Throw.*
*B.**	0 — 10 *cm.*	27°.8	*G.**	0 — 10 *cm.*	37°.3
*C.**	10 — 20 *cm.*	22°.1	*H.**	10 — 20 *cm.*	23°.5
*D.**	20 — 30 *cm.*	18°.0	*I.**	20 — 30 *cm.*	14°.5
*E.**	30 — 40 *cm.*	14°.2	*J.**	30 — 40 *cm.*	6°.0
*F.**	40 — 50 *cm.*	8°.5	*K.**	40 — 50 *cm.*	1°.0

☞ A figure representing the chords of the throws (see Table 3, column *d*) was here constructed by the student as explained in ¶ 189.

INFERENCES FROM THIS FIGURE.

l. Distance of north pole from end of magnet . . 19 *cm.*
m. Distance of south pole from end of magnet . . 14 *cm.*
n. { Distance between poles . . . . { 67 *cm.*
o. { The same in Exp. 75 (see *o*) . . . . { 66 *cm.*

* C. R. E., April, 1888.

EXPERIMENT LXXVII.

Throws of the needle of a ballistic galvanometer caused by revolving the coil of an earth-inductor . . . . 180°

A. About a horizontal axis.*			*B.* About a vertical axis.*		
	1.	55°.5		1.	17°.5
	2.	55°.		2.	17°.
	3.	56°.		3.	18°.
c.	Mean	55°.5	*d.*	Mean	17°.5

e. Chord of *c* 0.931 (Table 3). *f.* Chord of *d* 0.304 (Table 3).

g. Tangent of the angle of dip ($e \div f$) = ... 3.06
h. Angle of the magnetic dip (Table 3, column *b*, or Table 5) . . . . 71°.9

* C. R. E. April, 1888.

EXPERIMENTS LXXVIII.-LXXIX.

*A.** *Number of turns of wire in coil* 10

*B.** *Outside diameter of ring (mean of* 8 *obs.)* 37.936 *cm.*

*C.** *Depth of groove to outer surface of insulated wire (mean of* 8 *obs.)* . . . . 0.645 *cm.*

*D.** *Semi-diameter of wire (mean of* 4 *observations on* 10 *thicknesses)* . . . . . . 0.150 *cm.*

*E.** *Mean radius of coil* ($\frac{1}{2}B - C - D$) = 18.173 *cm.*

*f.** Constant of coil

$2\pi \times A \div E = 2 \times 3.1416 \times 10 \div 18.173 =$ 3.457

g. Reduction factor of the galvanometer, assuming a mean value 0.170 for the horizontal intensity of the earth's magnetism (Example 74 *u*); $0.170 \div f =$ 0.0492

h. The same for measurements in ampères ($10\,g$) = 0.492

Simultaneous deflections — mean of 3 *obs. in each case.*

I. Double-ring galvanometer.*		*J.* Single-ring galvanometer.*	
44°.00 N. E.	43°.83 S. W.	47°.53 E. S.	47°.13 W. N.
43°.97 N. W.	44°.33 S. E.	47°.53 E. S.	47°.13 W. N.
44°.03 N. W.	44°.47 S. E.	47°.50 E. N.	47°.93 W. S.
44°.10 N. E.	43°.97 S. W.	47°.53 E. N.	48°.00 W. S.
K. Zero reading.*		*L.* Zero reading.*	
0°.0 N. E.	—0°. 25 S. W.	0°.0 E. S.	— 0°.5 W. N.
*m.** Average deflection	44°. 09	*n.** Average deflection	47°.54
o. Tangent of *m* (Table 5)	0.9688	*p.* Tangent of *n.* (Table 5)	1.0928

q. Constant of the double-ring galvanometer,

$f \times o \div p =$ . . . . 3.06 +

r. Reduction factor of the double-ring galvanometer for ampères ($h \times p \div o$) = ... 0.555

* C. A. E., April, 1888.

NOTE. The same student found in March the value 0.1714 for the horizontal intensity of the earth's magnetism. Substituting this value instead of 0.170, the reduction factors in h and r become 0.496 and 0.559 +.

In the abbreviations above, the first letter refers to the pointer, the second to its deflection; thus the letters N. E. indicate readings of a pointer attached to the north end of a needle when deflected eastward; while the letters E. N. refer to observations of the east end of a (transverse) pointer when deflected northward.

EXPERIMENT LXXX.

A.	*Number of turns of wire in the large coil of the dynamometer*	100
B.	*Outside diameter of the coil*	27.00 *cm*
C.	*Inside diameter of the coil*	23.00 *cm.*
d.	Mean diameter, $\frac{1}{2}(B + C) =$	25.00 *cm.*
e.	Mean radius, $\frac{1}{2}d =$	12.50 *cm.*
f.	Constant of large coil $(2\pi e \div A) =$	50.27

G.	*Number of turns in small square coil*	79.5
H.	*Outside horizontal diameter*	5.09 *cm.*
I.	*Outside vertical diameter*	5.03 *cm.*
J.	*Width of* 80 *turns of wire*	4.80 *cm.*
k.	Mean diameter of wire, $(J \div 80) =$	0.06 *cm.*
l.	Mean horizontal diameter of coil $(H - k) =$	5.03 *cm.*
m.	Mean vertical diameter of coil $(I - k) =$	4.97 *cm.*
n.	Mean area of cross-section $(l \times m) =$	25.00 *sq. cm.*
o.	Magnetic area of coil $(G \times n) =$	1988 *sq. cm.*
p.	Constant of Dynamometer $(f \times o) =$	99937
q.	The same for ampères, or No. of dyne-centimetres due to one ampère, $(p \div 100) =$	999 +

R. Length of dynamometer wire subject to torsion 33.3 *cm.*

s. Coefficient of torsion of a wire 100 *cm.* long of the same size and material (Exp. 64) in *dyne-cm. per degree* . . . . 3.33 $\left\{\begin{array}{l}\textit{dy. cm.}\\ \textit{per deg.}\end{array}\right.$

t. Coefficient of torsion of dynamometer wire $s \times 100 \div R =$ . . . . 10.0 "

u. Reduction factor of the dynamometer for ampères $\sqrt{t \div q} =$ 0.100

V. Deflection of dynamometer (¶ 204) . . . 100°.0

W. Corresponding deflection of galvanometer 63°.4

x. Current through dynamometer,

$$u \sqrt{V} = 0.100 \sqrt{100.0} = 0.100 \times 10.0 =$$

1.00 ampères.

y. Tangent of the angle of deflection of the galvanometer (Table 5). Tan. $W.$ = tan. 63°.4 = 1.997

z. Reduction factor of the galvanometer $(x \div y) =$ 0.501

The same previously found (Exp. 78, *h*) . 0.492

Note. The value of the reduction factor (0.501) found in this experiment is the same as that which would have been found in Exp. 78 if the value 0.174 had been taken for the horizontal intensity of the earth's magnetism, instead of the value 0.170 found in Exp. 71–74.

EXPERIMENT LXXXI.

A. Weight of copper spiral before the experiment (mean of 3 *double weighings)* . . 10.945 *grams.*

B. Duration of the experiment, 31 *m.* 30 *sec.* = 1890 *sec.*

C. Deflections of the galvanometer at intervals of 1 *min.*

1	50°.7 *E. S.*	51°.2 *W.E.*	16	51°.0 *E.N.*	51°.5 *W. N.*
2	51.2	51.8	17	51.3	51.7
3	51.4	51.8	18	51.3	51.7
4	51.4	51.8	19	51.4	51.7
5	51.4	51.8	20	51.3	51.7
6	51.3	51.8	21	51.2	51.6
7	51.4	51.8	22	51.2	51.6
8	51.4	51.9	23	51.2	51.6
9	51.4	51.9	24	51.2	51.7
10	51.4	51.9	25	51.2	51.6
11	51.3	51.8	26	51.3	51.6
12	51.4	51.9	27	51.3	51.7
13	51.4	51.9	28	51.2	51.6
14	51.3	51.8	29	51.3	51.6
15	51.4	51.9	30	51.3	51.7
Mean	51.32	51.8[0]	*Mean*	51.25	51.64

D. Weight of copper spiral after the experiment (mean of 3 *double weighings)* . . 11.348 *g.*

e. Amount of copper deposited $(D - A)$ 0.403 "

f. Amount of copper deposited in 1 second $(e \div B) =$ . . . . 0.0002132 "

g. Amount of (cupric) copper deposited in 1 second by 1 absolute unit of current (Table 8) 0.00328 "

h. Amount deposited by 1 ampère $(\frac{1}{10} g) =$ 0.000328 "

i. Mean current in ampères $(f \div h) =$. 0.650 *ampères.*

j. Mean deflection, average of C_1 to $C_{30} =$ 51°.50 +

k. Tangent of this angle (Table 5) . . . . . . 1.257 +

l. Reduction factor of galvanometer $(i \div k) =$ 0.517

The same by dynamometer (Exp. 80, *z*) 0.501

The same by magnetic measurements (Exp. 78, *h*) 0.492

EXPERIMENT LXXXII.

☞ See Fig. 237.

	Number of vibrations observed.	Square of the No. of vibrations.	Apparent current in ampères.	The same allowing for earth's magnetism.
A. Vibrations completed in one minute by needle of vibration galvanometer under influence of earth's magnetism	9.0	81.	0.1	0.0
B. The same under the influence of currents from cells composed as follows (*see* ¶ 208) : —				
(1) *Zinc and copper strips in dilute sulphuric acid* . . .	50	2500	2.5	2.4
(2) *The same only zinc amalgamated*	50	2500	2.5	2.4
(3) *The same* 5 *minutes later than* (2)	45	2025	2.0	1.9
(4) *The same* 10 *minutes later than* (2).	39	1521	1.5	1.4
(5) *The same* 15 *minutes later than* (2).	81	961	1.0	0.9
(6) *The same after brushing bubbles off copper*	33	1089	1.1	1.0
(7) *The same after exposing copper* 5 *minutes to the air* . .	40	1600	1.6	1.5
(8) *The same after amalgamating copper*	40	1600	1.6	1.5
. . . . .				
(9) *The same with copper in porous cup containing a solution of sulphate of copper.*	35	1225	1.2	1.1
(10) *The same* 5 *minutes later than* (9)	35	1225	1.2	1.1
(11) *The same* 10 *minutes later than* (9)	36	1296	1.3	1.2
(12) *The same* 15 *minutes later than* (9).	37	1369	1.4	1.3

C. Weight of copper strip before obs. B 9 . . . . 50.0 *grams.*
D. Weight of zinc strip before obs. B 9 . . . . 50.0 "
E. Weight of copper strip after obs. B 12 . . . . 50.4 "
F. Weight of zinc strip after obs. B 12 . . . . 49.5 "
g. Weight gained by the copper in 15 minutes $(E - C) =$ . . . . 0.4 "
h. Weight lost by the zinc in the same time $(F - D) =$ . . . . 0.5 "

EXPERIMENTS LXXXIII.-LXXXIV.

A. Readings of Ammeter in ampères.	*B.* Readings of 1st galvanometer.*	*C.* Readings of 2d galvanometer.*	*d.** True current in ampères.	*e.* Correction of ammeter (*d* — *A*).
+ 0.10	0°.0	0°.0	0.00	— 0.10
+ 4.20	62°.0	65°.0	4.27	+ 0.07
+ 7.90	73°.3	76°.6	8.06	+ 0.16

* NOTE. The reduction factor of the 1st galvanometer with 10 turns of wire is about 0.50 (see Exp. 81); that of the second is 0.56 (Exp. 79); hence with only five turns of wire the reduction factors are 1.00 and 1.12 respectively. The current in *d* is accordingly 1.00 $\tan B + 1.12 \tan C$.

Readings of ammeter connected with different cells for different lengths of time: —

F. TIME.	*G. Bunsen cell.*	*H. Daniell cell.*	*I. Leclanché cell.*
0 minutes.	4.55 ampères.	2.00 ampères.	3.00 ampères.
5 "	4.50 "	2.10 "	2.80 "
10 "	4.45 "	2.18 "	1.00 "
15 "	4.20 "	2.20 "	
20 "	4.00 "	2.17 "	
25 "	3.80 "	2 05 "	
30 "	3.50 "	1.90 "	

See Fig. 237, page 470.

[NOTE. The results given above and in the figure were not founded upon actual observations, and are intended only to show how such observations should be made and represented.]

EXPERIMENT LXXXV.

Observations.

A. Weight of empty brass calorimeter* . 47.20 *grams.*
B. Weight of calorimeter with water* . 126.00 “

**Readings of galvanometer and thermometer at different times:* —

	C. Time.*			*D.* Galvanometer.*	*E.* Thermometer.*
1.	3 h.	14 m.	0 sec.	. . . .	23°.7
2.	“	15	0 “	0°.0 *E. S.*	
3.	“	16	0 “	. . . .	23°.8
4.	“	17	0 “	—0°.1 *W. N.*	
5.	“	18	0 “	. . . .	23°.8
6.	3 h.	19 m.	0 sec.	Circuit made	
7.	“	“	30 “	51°.1 *E. N.*	
8.	“	20	0 “	. . . .	24°.0
9.	“	“	30 “	48°.7 “	
10.	“	21	0 “	. . . .	24°.2.
11.	“	“	30 “	48°.0 “	
12.	“	22	20 “	. . . .	24°.6
13.	“	“	30 “	47°.5 “	
14.	“	23	0 “	. . . .	24°.8
15.	“	“	30 “	47°.0 “	
16.	“	24	0 “	. . . .	25°.0
17.	“	“	30 “	46°.8 *W. S.*	
18.	“	25	0 “	. . . .	25°.3
19.	“	“	30 “	46°.5 “	
20.	“	26	0 “	. . . .	25°.5
21.	“	“	30 “	46°.3 “	
22.	“	27	0 “	. . . .	25°.7
23.	“	“	30 “	46°.2 “	
24.	“	28	0 “	. . . .	25°.9
25.	“	“	30 “	46°.0 “	
26.	3 h.	29 m.	0 sec.	Current reversed	26°.0
27.	“	“	30 “	46°.3 *E. S.*	
28.	“	30	0 “	. . . .	26°.2
29.	“	“	30 “	46°.2 “	
30.	“	31	0 “	. . . .	26°.5
31.	“	“	30 “	46°.1 “	
32.	“	32	0 “	. . . .	26°.7

	C.* Time.			D.* Galvanometer.	E*. Thermometer.
33.	3 h.	32 m.	30 sec.	46°. *E. S.*	
34.	"	33	0 "	. . . .	26°.9
35.	"	"	30 "	46°. "	
36.	"	34	0 "	. . . .	27°.
37.	"	"	30 "	46°.2 *W. N.*	
38.	"	35	0 "	. . . .	27°.2
39.	"	"	30 "	45°.8 "	
40.	"	36	0 "	. . . .	27°.4
41.	"	"	30 "	45°.6 "	
42.	"	37	0 "	. . . .	27°.6
43.	"	"	30 "	45°.5 "	
44.	"	38	0 "	. . . .	27°.8
45.	"	"	30 "	45°.0 "	
46.	3 h.	39 m.	0 sec.	Current cut off	28°.0
47.	"	40	"	. . . .	28°.0
48.	"	41	"	. . . .	27°.9
49.	"	42	"	. . . .	27°.9
50.	"	43	"	. . . .	27°.9

* J. E. W., April, 1886.

CALCULATIONS.

f. Weight of water in calorimeter $(B - A) =$ 78.80 *g.*
g. Thermal capacity of calorimeter (¶ 90, 2; ¶ 91, III.), $0.094 \times A + 0.2 + 0.4 =$ . . . . 5.0 "
h. Total thermal capacity $(f + g) =$ . . . . 83.8 "
i. Rise of temperature observed $(E_{46} - E_6) =$ 4°.2 C.
j. Units of heat developed $(h \times i) =$ . . . . 352.
k. Length of time, $C_{46} - C_6 = 20$ *min.* 0 *sec.* = 1200 *sec.*
l. Units of heat per second $(j \div k) =$ . . . 0.293
m. Equivalent in watts (§ 15), $4.17 \times l =$. 1.22 *watts.*
n. Mean angle of deflection in *D* . . . . . 46°.64
o. Tangent of this angle (Table 5) 1.059
p. Reduction factor of the galvanometer with 5 turns of wire (see note, Example 84) 1.00
q. Current in ampères indicated $(o \times p) =$ . . . 1.059
r. Square of this current (Table 3, *C*), $q^2 =$ . . 1.122

s. Resistance of the conductor in ohms = power in watts necessary to maintain a current of one ampère (see § 136)
$= (m \div r) = 1.22 \div 1.122 =$ . . . 1.09 *ohms*.

NOTE. In the calculation above, only the first and last observations of temperature during the action of the current were utilized. The others would have given a somewhat larger result. The student's calculation gave 1.06 *ohms* as the resistance of the coil by the method of heating. The same student found the resistance of the same coil by comparison with B. A. units (May, 1886), to be 1.00 *ohms* (see note, Example 87). The probable error in determinations conducted as in the example is about 10 %.

EXPERIMENT LXXXVI.

FIRST METHOD.

A. *Deflection of a galvanometer with Bunsen cell and unknown resistance included in the circuit* . . . . 21°.6

B. *The same with* 7 *ohms in place of unknown resistance* . . . . 20°.0

C. *The same with* 6 *ohms in place of unknown resistance* . . . . 22°.0

d. Value of the unknown resistance by interpolation,
$6 + (C - A) \div (C - B) =$ . . . . 6.2 *ohms*.

SECOND METHOD.

E. *Deflection of differential galvanometer with unknown resistance in one circuit and* 6 *ohms in the other* . . . . — 6°.0

F. *The same with* 7 *ohms in place of* 6 *ohms* + 24°.0

g. Value of the unknown resistance by interpolation,
$6 + E \div (E - F) =$
$6 + [-6.0] \div [-6.0 - 24.0] =$ . . 6.2 *ohms*.

EXPERIMENT LXXXVII.

A. *Value of known resistance (said to be equal to 1.0132 B. A. units) in legal ohms (Table 50),*
1.0132 × 0.9889 = ... 1.0021 *ohms.*

*B.** *Distances of contact from end of bridge-wire nearer the unknown resistance measured:—*

(1) *from* 0 *cm. upwards (mean of* 6 *obs.)*	50.237 *cm.*
(2) *the same with current reversed (mean of* 6 *obs.)*	50.222 *cm.*
(3) *from* 100 *cm. downward (mean of* 6 *obs.)*	50.215 *cm.*
(4) *the same with current reversed (mean of* 6 *obs.)*	50.212 *cm.*
Mean	50.222 *cm.*

*C.** *Length of the bridge-wire* . . . . . . 100.00 *cm.*

d. Value of the unknown resistance,
$A \times B \div (C - B) =$
1.0021 × 50.222 ÷ 49.778 = ... 1.010 *ohms.*

* J. E. W., May, 1886.

Note. The value of the unknown resistance determined in this experiment includes that of the connecting wires, amounting to about 0.01 *ohm.* A deduction of 0.01 *ohm* should therefore be made in comparing this result with that obtained by the method of heating (Exp. 85).

EXPERIMENT LXXXVIII.

A. Value of known resistance used as a standard of comparison (see Example 87, *A)* 1.002 *ohms.*

B. Distance of contact on "bridge-wire" from end nearer unknown resistance (mean of 4 *observations — see B, Example* 87) . 50.00 *cm.*

C. Length of "bridge-wire" . . . . . . 100.00 *cm.*

D. Length of German silver wire (between connecting strips) constituting unknown resistance . . . . 100.00 *cm.*

E. Diameter of the wire (mean of 10 *obs.)* 0.05000 *cm.*

f. Cross section of wire with this diameter (Table 3, *G*) . . . . 0.001963 *sq. cm.*

g. Resistance of the wire $A \times B \div (C - B) =$ 1.002 *ohms.*

h. Resistance of a wire of the same material and diameter 1 *cm.* long $(g \div D) =$ . . 0.01002 *ohm.*

i. Resistance of a wire of the same material 1 *cm.* long and 1 *sq. cm.* in diameter $(h \times f) =$ 0.0000197 "

j. The same in microhms = specific resistance in microhms of a centimetre cube of the German silver 19.7

k. Compare (mean) value in Table 37 *a* . . 20.8

EXPERIMENT LXXXIX.

A. Value of known resistance used as a standard of comparison, 10 *B. A. units = in legal ohms* (*see Table* 50) 10 × 0.9889 = 9.889 *ohms.*

B. Distance of contact on "bridge-wire" from end connected with galvanometer, mean of 4 *observations* (*see Example* 87, *B*) . . 37.60 *cm.*

C. Length of "bridge-wire" . . . . 100.00 *cm.*

d. Resistance of galvanometer in legal ohms,
$A \times B \div (C - B)$ . . . . . . 5.959 *ohms.*

EXPERIMENT XC.

A. Value of known resistance used as a standard of comparison (*see Example* 87, *A*) 1.002 *ohms.*

B. Length of "bridge-wire" between point of contact and battery (*mean of* 2 *obs.*) . . 49.0 *cm.*

C. Length of "bridge-wire" . . . . 100.0 *cm.*

d. Resistance of battery in legal ohms,
$A \times B \div (C - B) =$ . . . . 0.96 *ohm.*

EXPERIMENT XCI.

FIRST METHOD.

*A.** *Deflection of Single Ring tangent galvanometer* . . . . 59°.5

*B.** *Corresponding deflection of Double-Ring tangent galvanometer with shunt (mean of 2 obs.)* . . . . 36°.05

*C.** *Resistance of the shunt* . . . . 0.10 *ohm*

d. Reduction factor of Single Ring galvanometer (Example 80, *z*) 0.50

e. Reduction factor of Double Ring galvanometer (Example 79, note) 0.56

f. Current through the Single Ring galvanometer, $d \times$ tangent of $A =$ 0.50×1.6977 (Table 5) $=$ . . 0.85 *ampère.*

g. Current through Double Ring galvanometer, $e \times$ tangent of $B = 0.56 \times 0.7279 = 0.41$ "

h. Current (by difference) through the shunt, $(f - g) =$ . . . . 0.44 "

i. Resistance of the galvanometer, $C \times h \div g = C \times 0.44 \div 41 =$ . . 0.11 *ohm.*

*F. S. D., May, 1888.

SECOND METHOD.

J. *Deflection of galvanometer through a large external resistance* . . . . 20°.0

K. *Value of this resistance* . . . . 1000 *ohms.*

L. *Deflection reduced by a shunt to* . . . . 10°.0

M. *Resistance of this shunt* . . . . 6.00 *ohms.*

n. Resistance of the galvanometer (¶ 224, 12)
$K \times M \times (J - L) \div (L \times K + L \times M - J \times M) =$
$1000 \times 6.00 \times 10.0 \div (10.0 \times 1000) + 10.0 \times 6.00 - 20.0 \times 6.00) =$
$60{,}000 \div 9940 =$ . . . . 6.043 *ohms.*

EXPERIMENT XCII.

*A.** *Deflection of Single Ring tangent galvanometer (mean of 2 obs.)* . . . . . . 62°.2

*B.** *The same with additional resistance* . . . 35°.25

*C.** *Value of resistance added* . . . . . . 2.00 *ohms.*

d. Resistance of battery, galvanometer, and connections (¶ 225, formula 10), $C \times$ tangent of $B \div$ (tangent of A — tangent of B) = (see Table 5) $2.00 \times 0.7067 \div (1.8967 - 0.7067)$ $= 2.00 \times 0.7067 \div 1.1900 =$ . . 1.10 + *ohms.*

e. Resistance of galvanometer (Example 91, *i*) 0.11 *ohm.*

f. Internal resistance of battery ($d - e$) = 0.99 "

* F. S. D., May, 1888.

NOTE. The electromotive force of the battery is found by multiplying the current in A ($0.50 \times$ tangent of A, see Example 80, z) and the total resistance in A, namely d. This gives $0.50 \times 1.8967 \times 1.10 = 1.04 +$ volts.

EXPERIMENT XCIII.

A. *1st resistance of the shunt* (*a c* Figs. 254 and 255) *such that a Daniell battery of* 2 *cells reduces the current from a single Bunsen cell to zero* . . . . 28.0 *ohms.*

B. *Corresponding resistance* (*b c*) *added to Daniell or main circuit* . . . . 0.0 "

C. 2*d resistance of shunt neutralizing the current in the Bunsen or side-circuit when a new resistance is added to the main circuit* . 168.0 *ohms.*

D. *Value of the resistance* (*in C*) *added to the main circuit* . . . . 10.0 "

e. Resistance of the Daniell battery (formula 6, ¶ 229),
$(A \times D - B \times C) \div (C - A) =$
$(28.0 \times 10.0 - 0) \div 140 =$ . . . 2.00 "

f. Average resistance of the Daniell cells ($\frac{1}{2}\,e$) = ... 1.00 "
Resistance of 1 Daniell cell by Mance's Method (Example 90, *d*) . . . . 0.96 "
The same by Ohm's method (Example 92, *f*) . . . . 0.99 "

NOTE. Taking the electromotive force of 1 Daniell cell as 1.04 (Example 92, note), that of 2 cells will be about 2.08 *volts.* This will give through an internal battery resistance 2.00 *ohms* (see *e*) and through an external resistance of 28.0 *ohms* (see *A*) a current of $2.08 \div (28 + 2) = 0.0693 +$ ampères. The fall of potential in passing through 28.0 *ohms* is accordingly $0.0693 \times 28.0 = 1.94$ *volts,* which must be equal to the electromotive force of the Bunsen cell.

EXPERIMENT XCIV.

First Part.

A. Electromotive force of a (given) Daniell cell from Exp. 92 (*see Example* 92, *note*) . 1.04 *volts*

B. Deflection of tangent galvanometer due to this and another Daniell cell in series . . 62°.4 N.E.

C. The same when the second cell is opposed to the first . . . . 1°.0 N. W.

D. Is the 2d cell stronger or weaker than the 1st? . . . . *Stronger.*

e. Electromotive force of stronger cell (see *D*) by formula 9, ¶ 231, and by Table 5,

$A \times (\tan. B + \tan. C) \div (\tan. B - \tan. C) =$

$1.04 \times (1.9128 + .0175) \div (1.9128 - .0175) =$

$1.04 \times 1.9303 \div 1.8953 \qquad = 1.06$ *volts.*

Second Part.

F. Deflection of tangent galvanometer due to 1 *Bunsen and* 2 *Daniell cells in series through an external resistance of about* 2 *ohms* 62°.0 N. E

G. The same when Bunsen cell is opposed to the two Daniell cells . . . . 5°.0 N. E.

H. Is the Bunsen cell stronger or weaker than the 2 *Daniell cells in series?* . . . . *Weaker.*

i. Electromotive force of the two Daniell cells $(A + e) =$ . . . . 2.10 *volts.*

j. Electromotive force of the Bunsen cell, by formula 8, ¶ 231 (see *D*) and by Table 5,

$i \times (\tan. F - \tan. G) \div (\tan. F + \tan. G) =$

$2.10 \times (1.8807 - .0875) \div (1.8807 + .0875) =$

1.92 *volts.*

EXPERIMENT XCV.

A.	*First resistance in thermo-electric circuit in addition to that of the thermo-element, of the galvanometer, and of the connecting wires*	0.0 *ohms.*
B.	*Corresponding resistance in circuit of Daniell cell, reducing the current to the same value as in A*	1313 "
C.	*Second resistance in thermo-electric circuit in addition to that of the thermo-element, of the galvanometer and of the connecting wires*	3.0 "
D.	*Corresponding resistance in circuit of Daniell cell, reducing the current to the same value as in C*	2563 "
e.	Increase of resistance in thermo-electric circuit corresponding to r, formula 10, ¶ 233 $(C - A) =$	3.0 "
f.	Increase of resistance in Daniell circuit, corresponding to $(R_2 - R_1)$ in formula 10, ¶ 233 $(D - B) =$	1250 "
g.	Electromotive force of (given) Daniell cell (Example 92, note)	1.04 *volts.*
h.	Electromotive force of thermo-element (formula 10, ¶ 233) $g \times e \div f = 1.04 \times 3.0 \div 1250 =$	0.0025 "

EXPERIMENTS XCVI.–XCVII.

	A. Distances between points of contact on uniform straight wire.*	*B.* Corresponding deflections of galvanometer with 3000 ohms added resistance.*
1.	10 *cm.*	7°.
2..	20 *cm.*	19°.
3.	30 *cm.*	31°.
4.	40 *cm.*	40°.
5.	50 *cm.*	47°.
6.	60 *cm.*	52°.5
7.	70 *cm.*	57°.
8.	80 *cm.*	61°.
9.	90 *cm.*	63°.5
10.	100 *cm.*	66°.

☞ See Fig. 260, page 527.

C. Deflection due to Daniell cell* . . . . . . 48°

D. Deflection due to Leclanché cell.* . . . . . 65°

E. Deflection due to Bunsen cell . . . . . . — — —

f. Number of *cm.* in *A* corresponding to a deflection in *B* of 48° (see *C*) by interpolation,
$50\ cm. + (48 - 47) \div (52.5 - 47) \times 10 =$ 51.8 *cm.*

g. Electromotive force of the (given) Daniell cell (Example 92, note) . . . . 1.04 *volts.*

h. Number of volts per *cm.* $(g \div f) = 0.020$ *volts per cm.*

i. Number of *cm.* corresponding to deflection 65° (see *D*) of Leclanché cell by interpolation,
$90\ cm. + (65 - 63.5) \div (66 - 63.5) \times 10 =$ 96.0 *cm.*

j. † Electromotive force of Leclanché cell
$(h \times i) =$ . . . . 1.9 + *volts.* †

k. Number of *cm.* corresponding to deflection of Bunsen cell . . . . ———

l. Electromotive force of Bunsen cell ($h \times k$) = ———

* S. L. B., May, 1888.

† This result (1.9 + *volts*) is far too great (see Table 35). The error was probably due to exhaustion of the Daniell cell used as a standard of comparison. A subsequent determination of one of the Daniell cells by Poggendorff's method (Exp. 99) gave 0.724 *volts* (H. F. B., May, 1888). Substituting this value for 1.04 *volts* in the example, the electromotive force of the Leclanché cell becomes 1.34 *volts*.

EXPERIMENT XCVIII.

*A.** *Distance between points of contact of poles of Leclanché cell (mean of* 5 *obs.)* . . 64.46 *cm.*

*B.** *Distance between points of contact of poles of Daniell cell (mean of* 5 *corresponding obs.)* . . . . 46.53 *cm.*

c. Ratio of the electromotive force of Leclanché to that of Daniell cell ($A \div B$) = . . . 1.385

d. Electromotive force of Daniell cell (Example 92, note) . . . . 1.04 *volts.*

e. Electromotive force of Leclanché cell ($c \times d$) = ... 1.42 "

* L. L. H., May, 1886

NOTE. The Leclanché cells used in the Jefferson Physical Laboratory (1885 to 1886) had electromotive forces 5 or 10 % higher than the value contained in Table 35.

EXPERIMENT XCIX.

A. Deflection of tangent galvanometer* . . . 45°.2

B. Resistance of the coil* (*Exps.* 85, 87) 1.002 *ohms.*

c. Reduction factor of galvanometer (Exp. 83) 1.00

d. Current in ampères ($c \times$ tangent of A),
(see Table 5) . . . . 1.007 *ampères.*

e. Electromotive force of the Daniell cell ($B \times d$) = ... 1.01 *volts.*
Previous determination (Exp. 92) . . 1.04 "

* F. S. D., May, 1888.

EXPERIMENT C.

Observations.	1st set of observations.	2d set of observations.
A. Mean difference in megadynes between the readings of two spring balances* .	0.108	0.123 *megadynes.*
B. Number of revolutions per second, reduced from obs. lasting* 100 *sec.* . .	2.65	2.27 *rev. per sec.*
C. Circumference of the pulley-wheel in centimetres*	30.3	30.3 *cm.*
D. Mean current in ampères indicated by (an ammeter or) tangent galvanometer.*	1.48	1.50 *ampères.*
E. Mean electromotive force in volts indicated (by a voltmeter or its equivalent.)*	6.3	6.2 *volts.*
Calculations.		
f. Power utilized by the motor in megergs per second ($A \times B \times C$) = .	8.7	8.5 *megergs per sec.*
g. The same in watts, $f \div 10$ = ...	0.87	0.85 *watts.*
h. Power spent upon motor in watts ($D \times E$) = ...	9.3	9.3 *watts.*
i. Efficiency of the electric motor ($g \div h$) $\times$ 100 %	9.4 %	9.1 %.

* F. S. D., May, 1888.

APPENDIX VI.

FIRST LIST OF EXPERIMENTS IN PHYSICAL MEASUREMENT, INTENDED TO COVER THE GROUND REQUIRED FOR ADMISSION TO HARVARD COLLEGE, BOTH IN ELEMENTARY AND IN ADVANCED PHYSICS.

NOTE. The experiments in this list are designated by the letter A. The abbreviations "H. U. Elem." and "H. U. Adv." refer to lists of experiments published by Harvard University, — the elementary list in October, 1889; the advanced list in June, 1890. The numbers following the abbreviations refer to the exercises in these lists to which a given experiment corresponds. Different parts of experiments are indicated by Roman numerals. Experiments marked "extra" do not correspond to any particular numbers in the lists, but are suggested as equivalents. A few experiments covering ground outside of the Harvard requirements are also included. If time is limited, these could naturally be left out, and are marked accordingly "Omit." The correspondence between this list and the two Harvard pamphlets is given at the end of the list.

Before performing the experiments the student should read the following sections in Part III.: In Chapter I., §§ 1 and 2; in Chapter II., §§ 23, 24, 30–33; in Chapter IV., §§ 50, 53–60.

HYDROSTATICS.

1 A. Find the length, breadth, and thickness in *cm.* of a block of wood, ¶ 3. Read § 5. Review §§ 1, 2, and 32, 1st paragraph. Calculate the volume in *cu. cm.* by multiplying the length, breadth, and thickness together.

Apparatus:—Block (wooden solid), and Gauge (vernier).
H. U. Elem., 7 I.

2 A. Find the weight in grams of the block used in 1 A, as in ¶ 2. Read §§ 6 and 9, also § 35. Calculate, as in ¶ 1, the density of the block.

Apparatus:—Balance (*b*); Block (wooden solid); Weights (*g*). H. U. Elem., 7 II.

3 A. Find the density of water, or better that of a saline solution of unknown strength, by loading a block of wood until it floats or sinks indifferently (foot-note page 2); then find, as in 1 A and 2 A, the volume, weight, and density of the block. The latter is equal to the density sought. Read §§ 62, 63, and 64.

Apparatus:—Balance (*b*); Block (hollow), Gauge (vernier); Weights (*g*); Lead, shot, and (salt) water.
H. U. Elem., 10, II.

4 A. Find the specific gravity of a block of wood by flotation in water. Mark the water-line in pencil at each corner, sighting (as in Fig. 6, page 10) the under surface of the water. Measure the *average* distances, d and d', of the water-line from the upper and lower surfaces of the block. Divide d' by $d + d'$, to find the specific gravity in question. Read §§ 3, 45, and 69. See Harvard Elementary List, Ex. 9, second part.

Apparatus:—Block (wooden, hollow); Metre rod, pencil, and water. H. U. Elem., 9, III.

5 A.* Find the weight required to sink a Nicholson's hydrometer to a given mark in water at, below, and above the temperature of the room (¶¶ 6 and 7). Plot a curve as in Fig. 7, page 12. Read § 4. Review § 59.

Apparatus: — Brush (camel's-hair); Nicholson's Hydrometer; Thermometer, and Weights (*cg*); Hot and cold water.

(H. U. Adv., 1.)

6 A.* Find with Nicholson's hydrometer the weight in air of some steel bicycle-balls, also that of a small wooden block, ¶ 8. Read § 43.

Apparatus: — Balls (steel); Block (small); Brush (camel's-hair); Nicholson's Hydrometer; Thermometer, and Weights (*cg*). H. U. Elem., 8 I., 9 I.

7 A.* Find with Nicholson's hydrometer the weight in water of objects used in 6 A (¶ 10), and calculate their apparent specific gravity (§ 66).

Apparatus same as in 6 A.

H. U. Elem., 8 II., 9 II.

NOTE. The blocks of wood must be held *down* by the lower pan (*l*, Fig. 9, page 15), since its weight in water is *negative*. Reverse the pan if necessary and place the block *under* it. The weight of water displaced by the block is the *difference* between the two weights required to sink the hydrometer with the block in air and in water. Divide the weight of the block by the weight of water it displaces to find its specific gravity.

* Exps. 5 A, 6 A, and 7 A may be performed with a Jolly (spring) balance instead of a Nicholson's hydrometer.

USE OF A BALANCE.

8 A. Find the sensitiveness of a balance with loads of 0, 20, 50, and 100 grams in each pan (¶¶ 20–21). Plot the results, Fig. 16. Read ¶ 22, §§ 25 and 26. Review §§ 30 and 33.

Apparatus: — Balance (*a*); Weights (*cg*).

H. U. Adv., 9.

9 A. Find the ratio of the arm of a balance (¶ 23). Repeat two or three times. Reduce as in ¶ 24. Read §§ 41 and 46. Estimate probable error (§ 50).

Apparatus: — Balance (*a*); Weight (*eg*).

H. U., Extra.

10 A. Find roughly the density of air as in Exp. XVII. (¶¶ 44 and 45), calculating the degree of exhaustion. Compare the observed density with data contained in Tables 19 [and 20] for the same conditions of pressure, temperature, [and humidity]. Read § 48.

Apparatus: — Balance (*b*); Barometer (aneroid); [Hygrodeik]; Pump (Richards); Rubber Stopper (1 hole); Specific Gravity Flask; Stopcock; Thermometer; Weights (*g*).

H. U. Elem., 11.

NOTE. Rough observations of the barometer, thermometer, and hygrodeik will suffice (see Exp. 5).

11 A. Find the density of some coal-gas as in Exp. XVIII. Calculate the density of air as in No. 10 A, from observations of a barometer, thermometer, and hygrodeik (¶¶ 13, 15). Read §§ 70 and 81; see Tables 18 *d* and *e*.

Apparatus: — Balance (*a*); Rubber Stopper; Barometer (aneroid); Hygrodeik; Specific Gravity Flask; Thermometer; Weights (*cg*), and Coal-gas.

H. U. Elem., Extra.

12 A. Find gross errors (if any) in the reading of a barodeik by comparing its indications with results obtained as in 10 A or 11 A. Employ the method of weighing by oscillations (¶ 20). Read §§ 49, 65, and 71.

Apparatus: — Balance (*a*), with Barodeik; Barometer (aneroid); Hygrodeik; Thermometer; Weights (*cg*).

H. U. Adv., 7.

13 A. Find the weight of a glass ball in air by a double weighing, ¶ 28. Weigh also a piece of cork coated with varnish. Read § 44. Reduce the results to vacuo by Table 21. Assume that the density of glass is 2.5.

Apparatus: — Balance (*a*); Ball (glass); Rings (small); Weights (*cg*). H. U. Adv., 8.

THE HYDROSTATIC BALANCE.

14 A. Find the weight of a glass ball in water (¶ 29). Read §§ 67 and 68. Calculate the volume and density of the ball.

Apparatus: — Arch (hydrostatic); Balance (*a*); Ball (glass); Beaker; Brush (camel's-hair); Stirrer; Thermometer; Weights (*cg*). Supplies: Wire and water.

H. U. Adv., 10, I.

15 A. Find the weight of the cork (in No. 13 A) in water by attaching a sinker to it, and weighing the sinker in water with and without cork (¶ 29). Calculate the density of the cork. Read § 34. Consider what assumptions you have made in this and in other experiments with the hydrostatic balance. Test the accuracy of one or more of these assumptions by weighing the cork in air *after* weighing it in water.

Apparatus: — Arch (hydrostatic); Balance (*a*); Beaker; Brush (camel's-hair); Cork; Sinker; Weights (*cg*). Supplies: wire and water. H. U. Adv., 12.

16 A. Find the weight of a glass ball (of No. 13 A) in alcohol at an observed temperature (¶ 30). Calculate the density of the alcohol (¶ 31).

Apparatus : — Arch (hydrostatic); Balance (*a*); Ball (glass); Beaker; Brush (camel's-hair); Stirrer; Thermometer; Weights (*cg*). Supplies: Wire and alcohol.

H. U. Adv., 11.

17 A. Find the readings of a densimeter in glycerine, water, and kerosene, and plot curve of corrections, as in Exp. XV., ¶¶ 39, 40, 41. Read § 36 (3).

Apparatus: — A Densimeter with jar containing glycerine, water, and kerosene. H. U. Adv., 16, I.

18 A. Find the density of three saline solutions by means of a densimeter, apply corrections found in 17 A. (Exp. XV., ¶¶ 39, 40, 41.)

Apparatus: — A Densimeter with three jars containing different saline solutions. H. U. Adv., 16, II.

19 A. Find the capacity of a capillary tube by means of mercury. See ¶ 169, II., and ¶ 170. Read § 39.

Apparatus: — Balance (*a*); Capillary Tube; Weights (*cg*); Mercury. H. U. Adv., 55, II.

NOTE. The student who wishes to take as little as possible for granted may himself determine the density of mercury, as in 21 A, before performing this experiment. The method described in 16 A is also (theoretically) possible, with, for instance, a platinum ball, which would sink in mercury. Attention is called to Tables 23 A, and 24, also to 23 B, which is intended especially to shorten calculations, in calibration by mercury.

20 A. Find the capacity of a Specific Gravity Bottle (¶ 32). Read ¶ 33.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*); Water.

H. U. Adv., 13.

21 A. Find the density of alcohol by the Specific Gravity Bottle, and calculate the strength of the alcohol (¶ 38). Use Table 27.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*); Alcohol.

H. U. [Elem. 10, I.] Adv., 15.

22 A. Find the volume of some steel balls, by the Specific Gravity Bottle. Calculate their density.

Apparatus: — Balance (*a*); Balls (steel); Specific Gravity Bottle (¶ 34); Stirrer; Thermometer; Weights (*cg*); Water. H. U. Adv., 14.

23 A. Find the volume of some crystals of sulphate of copper by the use of alcohol (¶¶ 36, 37), and calculate their density.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*). Supplies: Alcohol and crystallized sulphate of copper.

H. U., Extra.

24 A. Find the correction for one reading of a Vernier gauge (¶ 50, I). Read ¶ 47, but use Table 3, H. Read ¶¶ 48 and 49; also §§ 37, 72, and 73.

Apparatus: — Ball (glass); Gauge (vernier); Lens (magnifying). H. U. Adv., 2.

25 A. Find the pitch of a screw (¶ 50, II.).

Apparatus: — Balls (steel); Micrometer Gauge.

H. U. Adv., 3.

26 A. Find the constants of a spherometer (¶¶ 51 and 54).

Apparatus: — Ball (glass); Plate (glass); Spherometer.

H. U. Adv., 4.

27 A. Find the radii of curvature of 2 spherical surfaces, ¶ 55. Read ¶ 56.

Apparatus: — Lens (magnifying); Spherometer.

H. U., Extra.

Review Chapter V. (Hydrostatics).

PRESSURE.

28 A. Find the readings of a manometer under two or more different pressures (¶ 78). Find also the height of the barometric column (¶ 13). Read ¶ 77 and §§ 77, 78 and 79.

Apparatus: — Air Thermometer and Manometric Apparatus with mercury. H. U. Elem., 6.

29 A. Find the mercurial pressure required to keep air in manometer from expanding when heated from 0° to 100°. (¶ 76, as far as line 17, page 130.) Read §§ 74, 75, 76. Also ¶ 76. Calculate *e* by formula, page 131.

Apparatus: — Air Thermometer; Manometric Apparatus; Steam Boiler; Steam Jacket; Thermometer.

H. U. Elem., 25.

30 A. Find the fixed points of an Air-Thermometer (first paragraph, ¶ 73). Read § 80 and ¶ 74. Calculate *e* by formula X., page 126.

Apparatus: — Air Thermometer; Steam Boiler; Steam Jacket; Thermometer. H. U. Elem., 26.

31 A. Find the fixed, middle, and quarter points of a Mercurial Thermometer (¶¶ 66, 67, 68, 69, 70). Estimate tenths of a degree (§ 26).

Apparatus: — Beaker (for ice); Bunsen Burner; Steam Boiler; Thermometer. Supplies: Gas, ice, and water (or steam). H. U. Elem., 23 [Adv. 56].

32 A. Find the coefficient of expansion of water between about 20° and 100° (¶ 59). Read ¶¶ 60 and 61, and § 82. Review §§ 62 and 63.

Apparatus: — Expansion Apparatus with accessories. Supply of water and steam.

H. U. Adv., 53 [Elem., 10, III. or IV.].

33 A. Find the coefficient of expansion of alcohol from about 20° to 40° or 50° by the Specific Gravity Bottle. (¶¶ 62, 63).

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*). Supplies: Alcohol and hot water. H. U., Extra.

34 A. Find the coefficient of expansion of glass by the weight thermometer (¶ 240).

Apparatus: — Balance (*a*); Bunsen Burner; Steam Boiler; Steam Jacket; Thermometer (weight); Weights (*cg*). Supplies: gas, ice, mercury, and water (or steam). H. U. Adv., 58.

35 A. Find the coefficient of linear expansion of a brass rod from about 20° to 100° (¶ 57). Read § 83.

Apparatus: — Brass Rod; Micrometer Frame; Steam Boiler; Steam Jacket; Thermometer. H. U. Elem., 24.

36 A. Find the boiling-point of one or more liquids, and the melting-point of paraffine (¶¶ 83, 84).

Apparatus: — Stopper (1 hole); Test Tube; Thermometer. Supplies: Hot water, paraffine, alcohol, etc. H. U. Adv., 57.

37 A. Find the temperature of the air (¶ 15) and the dew-point (¶ 16). Read ¶ 17. Obtain the relative humidity (Table 14, A) and the pressure of aqueous vapor (Table 15).

Apparatus: — Cup (nickel-plated) and Thermometer, with ice and salt. H. U., Elem. 22, II.

38 A. Find the maximum pressure of aqueous vapor at about 40° (¶ 81).

Apparatus: — Balance (*b*); Rubber Stopper; Specific Gravity Flask; Thermometer; Weights (*g*) and hot water. H. U. Elem., 22, I.

CALORIMETRY.

39 A. Find different rates of cooling of a calorimeter (¶¶ 85, 87). Read ¶ 86, also §§ 47, 89.

Apparatus: — Calorimeter; Clock; Stirrer; Thermometer. Supply of hot water. H. U., Extra.

40 A. Find the thermal capacity of a calorimeter with thermometer and stirrer. ¶ 90 (1), I., and ¶ 90 (2); ¶ 91, I. and III. Read §§ 16, 84, 85. Review § 45.

Apparatus: — Balance (*b*); Calorimeter; Clock; Stirrer; Thermometer; Weights (*g*). Supply of hot water.

H. U. Adv., 60.

41 A. Find roughly the conductivity of sand by means of a calorimeter (¶ 241, I.).

Apparatus: — Balance (*b*); Calorimeter; Clock; Stirrer; Thermometer; Weights (*g*). Supply of sand and hot water. H. U., *omit.*

42 A. Find the specific heat of lead shot (¶ 94, I.). Read §§ 86 and 90. Use Formula VII., page 194.

Apparatus: — Balance (*b*); Bottle (ice water); Calorimeter; Thermometer; Weights (*g*); Shot, ice, and water.

H. U. Elem., 27 (Adv. 62).

43 A. Find the specific heat of alcohol or turpentine by the following electrical method: * place two equal (ohm) resistance-coils (Fig. 238, ¶ 212) in two equal calorimeters (see B., Fig. 239); fill one calorimeter with w' grams of water, the other with w'' grams of alcohol; pass a current from 2 Bunsen cells in series (§ 140) through both resistance-

* This experiment is taken from the Harvard University List of Advanced Physical Experiments, 1890, Exp. No. 64. It would be well in repeating it to interchange the contents of the two calorimeters (§ 44).

coils for about 10 minutes; note the rise of temperature (t') of the water and (t'') of the alcohol. Having found the thermal capacities (c' and c'') of the two calorimeters, as in ¶ 90, 2, we may calculate the specific heat of alcohol by the formula

$$s = \frac{wt' + c' t' - c'' t''}{w'' t''}$$

To obtain acccurate results by this method, an allowance for cooling must be made in estimating the temperatures in question.

Apparatus: — Balance (*b*); Battery (2 Bunsens); 2 Resistance-coils; 2 Stirrers; 2 Thermometers; Weights (*g*). Supplies: Alcohol and water, connecting wires.

H. U. Adv., 64.

44 A. Find the latent heat of liquefaction of water as follows: Mix 1 part, by weight, of ice with 5 parts of water at about 40° in a calorimeter. Note the temperature of the water *before* pouring it into the calorimeter, and after the ice has melted. Calculate the result by formula of ¶ 102, neglecting *c*. Read ¶ 102, also §§ 87, 88, 91.

Apparatus: — Balance (*b*); Shot Heater; Stirrer; Thermometer; Weights (*g*); ice and warm water.

H. U. Elem., 28 (Adv. 63).

NOTE. The object of this variation from the method of ¶ 101 is to avoid considering the thermal capacity of the calorimeter.

45 A. Find the latent heat of vaporization of water essentially as in ¶ 103; but find the temperature of the water by a single observation *before* pouring it into the calorimeter, and cut off the steam when the water reaches the temperature of the room. (See note under 44 A.) Calculate the result by the formula of ¶ 104, *neglecting c*. Read ¶ 104.

Apparatus: — Balance (*b*); Steam Boiler; Steam Trap; Stirrer; Thermometer; Weights (*g*); Ice and warm water.

H. U. Elem., 29.

46 A. Find the heat of combination of zinc and nitric acid (¶¶ 105, 1, 106).

Apparatus: — Balance (*a*); Calorimeter, with glass lining; Clock; Stirrer; Thermometer; Weights (*cg*). Supplies: Zinc filings and dilute Nitric Acid.

H. U., *omit.*

47 A. Find the heat of combination of zinc oxide and nitric acid. (¶¶ 105, 2, 106.)

Apparatus same as in 46 A. Supplies: Zinc Oxide and dilute Nitric Acid. H. U., *omit.*

Review Chapter VI.

RADIANT HEAT AND LIGHT.

48 A. Find the candle-heat power of a kerosene lamp (¶¶ 111, 112), and calculate that of a lamp burning 8 grams of kerosene per hour (¶ 113). Read §§ 94, 95, 148.

Apparatus: — Balance (*b*); Candle; Clock; Galvanometer (astatic); Kerosene Lamp; Optical Bench; Thermopile: Weights (*g*). H. U. Adv., 99.

49 A. Find the candle-power of a kerosene lamp by Bunsen's photometer (¶ 114, I). Read ¶ 109. Reduce the candle-power of the lamp to 8 grams per hour. Use formula and reasoning of ¶ 113.

Apparatus: — Candle; Kerosene Lamp; Optical Bench; Photometer. H. U. Elem., 34 (Adv. 32).

50 A. Find the principal focal length of a lens by two different methods (¶ 116 (1), (2)). Read § 103.

Apparatus: — Chimney (perforated); Kerosene Lamp; Lens (magnifying); Optical Bench.

H. U. Elem., 36.

51 A. Find the equivalent focal length of a compound lens as follows: Place two lights at two points H' and H'' (Fig. 7, § 104) as far as possible from the lens, and separate them so as to produce the greatest measurable distance between the images B' and B''. Measure this distance and call it d. Now substitute the lens from 50 A; focus by *moving the screen*; and let the new distance between the images be d'.

Calculate the focal length (F) of the compound lens from that (F') of the lens in 50 A, by the formula —

$$F = F' \times \frac{d}{d'}.$$

Read first two paragraphs of § 104. See Harvard List of Advanced Physical Experiments, No. 45.

Apparatus: — Candle; Kerosene Lamp; 2 Lenses ("doublet" and magnifying lens); Metre Rod; Optical Bench.

H. U. Elem., 38 (Adv. 45, 46).

52 A. Find several conjugate focal lengths of a lens (¶ 117, (1), (2), and (3)). Note the size of the images (see § 104). Calculate the principal focal length of the lens. Use formula, page 238.

Apparatus: — Chimney (perforated); Kerosene Lamp: Lens (magnifying); Metre Rod; Optical Bench.

H. U. Adv., 42 (Elem., 37, I.).

53 A. Find the virtual foci of several (nearly) plane mirrors (¶ 118). Tell which are convex and which concave, remembering that the virtual images (§ 104) of *convex* mirrors are *nearer* than the objects producing them. Read ¶ 118.

Apparatus: — Mirrors (small); Optical Bench.

H. U. Elem., 35 (Adv. 41).

54 A. Find 3 virtual foci of a long-focus converging lens (¶ 119, II.). Calculate the principal focal length.

Apparatus :— Lens (long focus), and Optical Bench.
H. U. Elem., 37, II.

55 A. Find the zero-reading (§ 32) of a sextant (¶ 123). Read § 97.

Apparatus :— A Sextant. H. U. Adv., 35, I.

56 A. Find by a sextant the angular semidiameter of the sun (¶ 124, I.).

Apparatus :— A Sextant. H. U. Adv., 35, II.

57 A. Find the three angles of a prism (¶ 125).

Apparatus :— A small Prism ; Kerosene Lamp (with slit); Spectrometer (or sextant). H. U. Adv., 50.

58 A. Find the angle of minimum deviation for a ray of sodium light passing through a prism angle of known magnitude (¶¶ 126, 127). Read ¶ 128 and § 102.

Apparatus.— Prism (used in 57 A) ; Sodium Flame (with slit) ; Spectrometer (or sextant). H. U. Adv., 52.

59 A. Find the distance between the lines of a diffraction grating (¶ 130). Read ¶ 129, § 101.

Apparatus :— Diffraction Grating ; Sodium Flame (with slit) ; Spectrometer (or sextant). H. U., Extra.

SOUND (§§ 92–96).

60 A. Find the wave-length of sound from a tuning-fork in a rubber tube (¶ 131, I.). Read § 100.

Apparatus :— Metre Rod ; Rubber Tube ; Tuning-Fork *Y* tube. H. U. Elem., 32.

61 A. Find the wave-length of sound from a tuning-fork in a resonance tube (¶ 132). Read §§ 98 and 99. Notice

that the lengths of the tube corresponding to a given fork are nearly proportional to the *odd* integers 1, 3, 5, &c.

Apparatus: — Resonance Tube and Tuning-Fork (A = 220). H. U. Adv., 26.

62 A. Find the pitch of a tuning-fork by the graphical method (¶ 139). Read §§ 7 and 96.

Apparatus: — Smoked Glass app.; Tuning-Fork ($c = 64$). H. U. Elem., 31.

63 A. Find the pitch of a tuning-fork by the toothed wheel (¶ 144). Read ¶ 145.

Apparatus: — Toothed Wheel apparatus; Tuning-Fork ($c = 64$). H. U., Extra.

64 A. Find the musical interval between two tuning-forks by means of a monochord (¶ 133, III.). Read ¶ 134.

Apparatus: — A Monochord and 2 Tuning-Forks (A = 216 to 220, $c = 256$). H. U. Adv., 24.

Note. A musical ear is of service in making rapidly the necessary adjustments of a monochord, but is not absolutely necessary for this experiment. Unison between the fork and string may be tested by touching the base of the fork to the end of the string. If unison exists the fork should communicate its vibration to the string.

If l is the length of the string and m its mass per unit of length, the number of vibrations (n) produced in one second by a stretching force (f) in dynes (equal to wg if w is the stretching weight) may be found by the formula —

$$n = \frac{1}{2l}\sqrt{\frac{f}{m}}.$$

Students not preparing for Harvard College may substitute ¶ 133, I. or II. for ¶ 133, III.

65 A. Find by Lissajous' curves (¶ 143) the musical interval between two C–forks 2 octaves apart, also find the

musical interval between the higher of these forks and a $G\sharp$ fork two "octaves" and a "third" below it. Read ¶¶ 134 and 142.

Apparatus :— Lens (small) ; 3 Tuning-Forks ($C = 256$, $C = 64$, $G\sharp = 51.2$) ; Kerosene Lamp for smoking, and sealing wax. H. U. Adv., 29.

NOTE. Instead of the forks mentioned above, two A-forks and a D-fork may be used ($A = 216$, $A = 54$, $D = 72$) or only 2 forks ($C = 64$, $C = 128$), as suggested in ¶ 143. The advantage of using three forks is that the labor and apparatus required in the next experiment may be greatly reduced.

66 A. Find the pitch of a set of forks, covering a known musical interval by the method of beats (¶ 141). Read ¶ 140.

Apparatus : — A clock and 5 tuning-forks, $G\sharp = 51.2$, $A = 54$, $A\sharp = 57$, $B = 60$, $C = 64$.

H. U. Adv., 25.

NOTE. If in the last experiment (No. 65 A.) an A- and a D-fork were used, 6 forks will now be required, namely : $A = 54$, $A\sharp = 57$, $B = 60$, $C = 64$, $C\sharp = 68$, and $D = 72$. If only two forks were used ($C = 64$ and $C = 128$), a set of 17 forks will be necessary to cover the interval in question.

The results of the last experiment (No. 65 A.) are reducible to the form (see ¶ 142, formula I.),

$$P = n_1 p_1 + c \ (1), \text{ and } P = n_2 p_2 + c_2 \ (2) ;$$

hence, substracting (2) from (1), we have

$$n_1 p_1 - n_2 p_2 = c_2 - c_1 \ (3).$$

Now from this experiment (No. 65 *A*) we find

$$p_2 - p_1 = p \ (4); \text{ whence } n_2 p_2 - n_2 p_1 = n_2 p \ (5).$$

Adding (3) and (5) we have finally

$$(n_1 - n_2)\, p_1 = n_2 p + c_2 - c_1 \ (6),$$

where n_1 and n_2 represent the respective numbers of lobes visible when the first and when the second of two forks are compared with a third fork higher than either of them; p_1 the pitch of the first fork, p the excess of the second fork over the first, $c_2 - c_1$ the algebraic excesses of the third fork over the nearest harmonic of the first and second, respectively. The pitch of the forks chosen above is such that $n_2 - n_1 = 1$. If they are carefully tuned or loaded, c_1 and c_2 may be made nearly equal or both very small, so that in either case $c_2 - c_1$ may be neglected. After any such adjustment of pitch the observations named in Nos. 65 and 66 must of course be repeated.

67 A. Find the pitch of the note due to longitudinal vibrations in a wire (¶ 248 I) either by a pitch-pipe (Fig. 273), or (in the absence of a musical ear) by a resonance tube, ¶¶ 132, 134, II. Calculate the velocity of sound in the wire (¶ 248).

Apparatus: — A Pitch-Pipe (or Resonance Tube); Tape Measure: Wires; Cloth, Resin, etc.

H. U. Adv., 27.

68 A. Find the pitch of the note due to torsional vibrations in a wire (¶ 248, II.), either by a pitch-pipe or by a resonance tube. Calculate the velocity of these torsional vibrations in the wire.

Apparatus: — Same as in 67 A.

H. U., Extra.

69 A. Find the velocity of sound (¶ 135 (1), (2), & (3); ¶ 136, first paragraph; ¶ 137, III.). Read ¶¶ 138, 135

(4); also §§ 8, 10, 92 and 93. Use formula II, page 281.

Apparatus: — Clock; Signalling Apparatus; Tape Measure. H. U. Elem., 30.

Review Chapter VII

VELOCITY.

70 A. Find the velocity (v) of a bullet by a ballistic pendulum (¶ 147, (7)) as follows: Find the weight (m) of the bullet and that (M) of the pendulum; measure the length ($A\ C$) of the suspending cords. Project the bullet into the pendulum. Let the pendulum be caught by a ratchet at its furthest point. Measure the distance (*A. B.* Fig. 9, § 109) through which it has swung. Read §§ 11, 12, 106 and 109. Calculate the velocity V of the pendulum by the formula

$$V = A\,B\sqrt{\frac{980}{A\,C}} \text{ (see § 109.)}$$

Now read §106. The impulse ft which the bullet gives the pendulum may be measured either (1) by the momentum lost by the bullet, that is, $m\ (v - V)$, or (2) by that gained by the pendulum ($M\,V$); hence $M\ V = m\ (v - V)$, or

$$v = \frac{m + M}{m} V \text{ (see ¶ 147, 7).}$$

Apparatus: — Ballistic Pendulum; Bullet (with means of projecting it); Clock; Metre Rod.

H. U. Elem., Extra.

71 A. Find the average velocity of a falling body (¶ 148). Read §§ 107, 108 and 111. Calculate the acceleration of gravity.

Apparatus: — Clock; Falling Body Apparatus; Metre Rod. H. U. Adv., 18.

72 A. Find the length of a seconds, $\frac{1}{2}$ seconds and $\frac{1}{4}$ seconds pendulum (¶ 149). Tabulate results as on page 319. Read §§ 28, 29, 40, 61, and 110.

Apparatus :— Clock ; Metre Rod ; Pendulum (simple).
H. U. Elem., 19 (Adv., 17).

73 A. Find the relative masses of two billiard balls as suggested on pages 312–313. Make a series of experiments all performed in exactly the same manner. Have a metre rod fixed in position, at one time so as to measure the distance $A\,A''$, at another time the distance $B\,B''$, etc.

Apparatus : — Balls (billiard) ; Metre Rod.
H. U. Elem., 20.

74 A. Find the mass of a lead bullet by the method of oscillations (¶ 154). Read ¶ 155.

Apparatus : — Clock, Spiral Spring Apparatus ; Weights (*cg*) and lead bullet. H. U. Elem., 18.

FORCE AND ELASTICITY.

75 A. Find the weight in kilograms of a 28-lb. weight, (¶ 159, 1) ; a 56-lb. weight (¶¶ 159, 2 and 159, 3), and a 4-lb. weight (¶ 159, 4), using a lever and 1 or 2 spring balances of 10 kilos capacity.

Apparatus : — Balances (spring, 10 *k.*) ; Lever ; Weights (safety-valve) with cords. H. U. Elem., 14.

76 A. Find with 1 or 2 spring balances of 10 kilograms capacity and a system of cords, the weight in kilograms of a 4-lb. weight (¶ 159, 5), and of a 56-lb. weight (¶ 159, 6). Read § 105.

Apparatus : — Balances (spring 10 *k.*) ; Weights (safety valve) with cords. H. U. Elem., 12.

77 A. Find the weight of a board, as in ¶¶ 160 and 161. Read § 112.

Apparatus: — Plank (1 × 6 ft.); Pendulum (simple); Triangular supports; Weights (safety valve).

H. U. Elem., 17.

78 A. Find the stiffness of 5 beams by bending them (¶ 162). Read § 115.

Apparatus: — Beam (steel) Micrometer; Triangular supports; Weights (*kg*). H. U. Elem., 3.

79 A. Find the (torsional) stiffness of 2 or more rods by twisting them (¶ 164). Read §§ 13, 113 and 116.

Apparatus: — Balance (spring, 10 *k.*) and Torsion Apparatus. H. U. Elem., 4.

80 A. Find the coefficient of torsion of wire by a torsion balance (¶ 165). Review § 116.

Apparatus: — Gauge (micrometer); Metre Rod; Torsion Balance; Torsion Head; Weights (*cg*).

H. U. Elem., 15.

81 A. Find Young's Modulus of Elasticity for a wire (¶ 167). Read § 114.

Apparatus: — Gauge (micrometer); Micrometer (electric); Tape measure; Weights (*kg*); Young's Modulus Apparatus. H. U. Elem., 2 (Adv., 54).

82 A. Find the breaking strength of several wires (first paragraph, ¶ 168). Weigh a known length of the wire, and calculate what length would break under its own weight. Read ¶ 168.

Apparatus: — Balance (spring, 10 *k.*); Bobbins and Wires. H. U. Elem., 1.

83 A. Find the surface tension of water by means of the capillary tube of No. 16 A (¶ 169, II.). Read ¶ 170.

Apparatus: — Beaker; Capillary Tube; Metre Rod; Thermometer. H. U. Adv., 55, II.

84 A. Find by two methods the coefficient of friction of wood on wood (¶ 171, I., II.). Review § 105.

Apparatus: — Balance (spring, 10 *k.*); Board and Plank; Weights (*kg*). H. U. Elem., 13.

85 A. Find the efficiency of a pulley (1) for raising heavy weights and (2) for multiplying motion (¶ 173). Read §§ 14 and 117.

Apparatus: — Balance (spring, 10 *k.*); Metre Rod; Tackle; Weights (safety-valve). H. U. Elem., 21.

86 A. Find the efficiency of a Water Motor (¶ 174). Read ¶ 175, also §§ 15, 118.

Apparatus: — Balance (rough); Clock; 2 Spring Balances; Jar; Tape Measure; Water Motor (with pressure gauge) weights (*kg*). H. U., *omit.*

87 A. Find (roughly) the mechanical equivalent of heat by means of lead shot (¶ 177, first paragraph). Read ¶¶ 176 and 178.

Apparatus: — Paste-board Tube (with corks); Thermometer, and some Lead Shot. H. U. Adv., 65.

Review Chap. VIII., as far as § 119. Read §§ 119–122.

MAGNETISM.

88 A. Find the distance between the poles of a magnet by means of iron-filings, and confirm by a small compass-needle (¶ 179). Read §§ 126 and 127.

Apparatus: — Compass (vibrating); Magnet (compound); Iron Filings; Photographic paper and pencil.

H. U. Elem., 40.

89 A. Find the attraction and repulsion between two parallel magnets at a given distance (¶ 180). Estimate the strength of the poles (¶ 181). Read §§ 17 and 129.

Apparatus: — Balance (*a*); 2 Blocks (*cu. cm.*); Gauge (vernier); 3 Magnets (compound); Weights (*cg*).

H. U., Extra.

Note. In this and in following experiments, the distance between the poles of the (short) compound magnets may be called equal to $\frac{7}{10}$ the length of the magnet (see ¶ 179).

90 A. Find the couple exerted by the Earth's Magnetism upon magnets by means of torsion (¶ 182). Estimate "H." Read § 128.

Apparatus: — 3 Magnets (compound); Torsion Head and Wire tested in No. 80, A.; Wax, and Pins to serve as sights.

H. U. Adv., 68, I.

91 A. Find the deflection of a compass-needle due to a magnet of known strength (from No. 89 A) at a given distance (¶ 183). Read ¶¶ 184 and 185. Estimate "H." Calculate the true value of "H" from the estimates in Nos. 90 A and 91 A.

Apparatus: — Compass (surveying); 3 Magnets (compound); Metre Rod. H. U. Adv., 68, II.

92 A. Find the distribution of magnetism on a magnet by the method of vibrations (¶ 186). Plot a curve (Fig. 205). Estimate the distance between the poles.

Apparatus: — Clock; Magnet (vibrating needle); Magnet (long-bar); Metre Rod; Test-tube. H. U. Adv., 66.

93 A. Find the distribution of magnetism on a magnet by means of an induction coil (¶ 189). Plot the curve and estimate the distance between the poles as in No. 92 A. Read § 147, also ¶¶ 187 and 188.

Apparatus: — Galvanometer (astatic); Helix (sliding); Magnet (long-bar); Metre Rod. H. U. Adv., 69.

94 A. Find the magnetic dip by the Earth-Inductor (¶ 192), and confirm by means of a dipping needle. Read ¶¶ 190 and 191. Review § 128.

Apparatus : — Earth-Inductor ; Galvanometer (astatic, loaded so as to answer for a ballistic galvanometer), and a Level. H. U. Adv., 70.

ELECTRICAL CURRENT MEASURE,
§§ 18, 19, 130, 131.

95 A. Find the relative strength of battery currents from a 1-fluid cell under given conditions (¶ 208, (1) to (8)). Read §§ 123, 124, and ¶ 207. Reduce results as in ¶ 209, and plot them as in Fig. 237.

Apparatus : — Battery (1 Daniell) ; Compass (vibrating) ; Galvanometer. (The porous cup is to be removed from the Daniell cell.) H. U. Elem., 41.

96 A. Find the deflections of a tangent compass at the centre of a coil of wire due to currents from a Daniell cell under the conditions of ¶ 208 (9) to (12). Plot the results as in 95 A. Weigh the zinc and the copper before and after the experiment, and calculate the gain or loss of weight in each case. Read § 144. Review ¶ 209.

Apparatus : — Balance (*b*) ; Battery (1 Daniell) ; Compass (surveying) ; Galvanometer ; Weights (*g*).

H. U. Elem., 42.

97 A. Find the constant and reduction factor of a Single-Ring Tangent Galvanometer (¶¶ 198, 199, formulæ (5) and (6)). Read §§ 18, 19, 132 and 133.

Apparatus : — Battery (6 Daniell) ; Galvanometer (S. R.), and connecting wire. H. U. Adv., 71, I.

98 A. Find the reduction factor of a Double-Ring Galvanometer by the method of comparison (¶ 201). Read ¶ 200.

Apparatus : — Battery (2 Daniell) ; 2 Commutators ; 2 Galvanometers (S. R. and D. R.), and connecting wire.

H. U. Adv., 73.

99 A. Find the reduction factor of an Astatic Galvanometer by the method of comparison (¶ 201), as follows: Connect the astatic galvanometer in series with a rheostat of several thousand ohms resistance, a tangent galvanometer, and a battery. Arrange a shunt of about 1 ohm resistance so as to cut out the rheostat and astatic galvanometer. Change the resistances of the shunt and rheostat so that both galvanometers may give measurable deflections (e. g. 45°. Read § 38). Note what plugs are removed from the rheostat, also the length, diameter, and material of the shunt. Calculate the reduction factor of the combination as in the last experiment (No. 98 A).

Apparatus: — Battery (1 Daniell); 2 Galvanometers (astatic and D. R.); Gauge (micrometer); Metre Rod; Resistance Box; 1 Metre of German silver wire (about No. 25 B. W. G.). H. U. Adv., 86.

NOTE. If R, G, and S are the respective resistances of the Rheostat, Galvanometer, and Shunt, and if I is the reduction factor of the combination, the reduction factor (i) of the astatic galvanometer alone is —

$$i = I \times \frac{S}{R + G + S}.$$

The Galvanometers should be marked and the shunt laid aside for Exps. No. 101 A and 108 A, respectively; or the whole experiment (No. 99 A) may be deferred until G and S have been determined.

100 A. Find the reduction factor of a Dynamometer by comparison with a Single-Ring Galvanometer (¶ 204). Read ¶ 202, § 131.

Let C be the current in ampères indicated by the galvanometer, and a the angle of torsion in the dynamometer; then we find the reduction factor D by the formula —

$$D = \frac{C}{\sqrt{a}}.$$

Apparatus: — Battery (3 Bunsen or 6 Daniell); 2 Commutators; Dynamometer; Galvanometer (S. R.), and connecting wires. H. U. Adv., 98, I.

101 A. Find by measurement the reduction factor of a Dynamometer (¶ 203). Read §§ 134 and 135.

Use the formula

$$D = 10\sqrt{\frac{t}{KA}}.$$

Calculate the current C in No. 100 A by the formula

$$C = D\sqrt{a};$$

then find I and H, as in ¶ 204.

Apparatus: — Dynamometer; Gauge (vernier, long); Torsion Balance, and Weights (*cg*). H. U. Adv., 98, II.

102 A. Find the reduction factor of a galvanometer by the electro-chemical method (¶ 205). Calculate "H" (¶ 206). Read §§ 142 and 143.

Apparatus: — Balance (*a*); Battery (Daniell); Clock; Commutator; Galvanometer (S. R.); Weights (*cg*) and a spiral of copper wire. H. U. Adv., 71, II.

Review Chapter IX., omitting § 124.

ELECTRICAL RESISTANCE.

103 A. Find the electrical resistance of a coil of wire by the method of heating (¶¶ 212, 213). Read §§ 20, 136, and 137.

Apparatus: — Balance (*b*); Battery (2 Bunsen); Calorimeter; Resistance-Coil; Stirrer; Thermometer; Weights (*g*). H. U. Adv., 78.

104 A. Find the length of copper wire about $\frac{1}{4}$ mm. in diameter (No. 31 B. W. G.), which can be substituted for a

1-ohm coil (C) in the circuit of a Daniell cell (B) and galvanometer (G) — see Fig. 243, page 476 — without changing the deflection. Repeat with a double wire, with a German silver wire of the same diameter, and with one of twice the diameter, or 4 times the cross section (about No. 25 B. W. G.). Read ¶ 218, also § 140.

Apparatus: — Battery (1 Daniell); Compass (surveying); Galvanometer; Resistance-Coil (1 ohm); and wires as stated.

H. U. Elem., 44 (Adv., 76).

105 A. Find the (external) resistance of a circuit, as follows: First, note the deflection of the galvanometer due to each one of two equal cells, then join the cells in series (Fig. 20, § 146), and include German silver wire enough in the circuit to give the same (average) deflection as before.

Apparatus: — Battery (2 Daniell); Compass (surveying); Galvanometer with German silver wire.

H. U. Elem., 45, I. (Adv. 77, I.).

Proof. Since the electromotive force is doubled (§ 146) and the current is the same, the total resistance must be doubled. Now the internal resistance (§ 140) is doubled, hence the external resistance must also be doubled. The resistance added is accordingly equal to the original external resistance.

106 A. Find the electrical resistance of a conductor by means of a differential galvanometer (¶ 216).

Apparatus: — Battery (1 Daniell); a Galvanometer (astatic with differential connections); the Helix of No. 93 A; a Key; and a Resistance-Box. H. U. Adv., 85.

107 A. Find gross errors (if any) in a resistance-box by means of a Wheatstone's Bridge (¶ 217). Use as a (rough) standard of comparison the resistance-coil tested in No. 103 A. Select a resistance-box in which no *gross* errors are discov-

ered, and assume in future that the resistances are accurate. Read §§ 42 and 141.

Apparatus: — B. A. Bridge; Battery (1 Daniell); Galvanometer (astatic); Resistance-Box and Resistance-Coil.

H. U. Adv., 81.

108 A. Find by Wheatstone's Bridge the resistance of the shunt used in No. 99 A, ¶ 217, and calculate the specific resistance of the material of which it is made (¶ 218). Read ¶ 219.

Apparatus: — A B. A. Bridge; Battery (1 Daniell); Galvanometer (astatic); and Shunt.

H. U. Adv., 82.

109 A. Find the resistance of a galvanometer by Thomson's method (¶ 220). Read ¶ 221.

Apparatus: — Same as in Exp. 108 A, plus a magnet.

H. U. Adv., 90.

110 A. Find the Resistance of a battery by Mance's method (¶ 222). Read ¶ 222 *a*.

Apparatus: — A B. A. Bridge; a Battery (1 Daniell); a Galvanometer (astatic); a Key; a Magnet (compound small); and a Resistance-Box. H. U. Adv., 89.

111 A. Find the mean resistance of a Daniell cell as follows: Note the deflection of each of two cells as in 105 A, and join them in multiple arc (Fig. 19, § 146). Include in the circuit enough German silver wire to give the same average deflection as before. Calculate the resistance of this wire, and multiply it by 2 to find the resistance sought.*

Proof. Since the current and electromotive force are unchanged (§ 146) the total resistance is unchanged (§ 138). The resistance added is therefore equal to the decrease in the

* It is not necessary to cut the wires in 106 A and 111 A. A greater or less length may be included between two clamps, as in ¶ 237. The wires should be kept straight, as in Fig. 249, page 486.

battery resistance caused by arranging the cells in multiple arc. Now this is half the resistance of a simple cell, therefore, etc.

Apparatus : — Battery (2 Daniell) ; Compass (surveying) ; Galvanometer, clamps and wire,

H. U. Elem. 45, II. (Adv. 77, II.).

112 A. Find the resistance of a battery by Ohm's method (¶ 225). Read § 138.

Apparatus : — A Battery (1 Daniell) ; Galvanometer (S. R.) and Resistance-Box. H. U. Adv., 75.

NOTE. The battery cell should be marked so that it can be identified later on.

113 A. Find the resistance of a battery by Thomson's method as follows : Connect a Daniell cell (*B*, Fig. 253, page 499) with an astatic galvanometer (*G*), through a resistance box (*R*), with enough plugs removed to reduce the deflection of the galvanometer to about 45°. Now connect the poles of the battery with a shunt (*S*) (of about 1 ohm's resistance), and find what resistance (*r*) in the galvanometer circuit will give the same deflection as before. Calling the respective resistances of the Resistance-Box, Galvanometer and Shunt, *R*, *G* and *S*, we find the battery resistance by the formula

$$B = S\,\frac{R - r}{r + G}.$$

Apparatus : — Battery (1 Daniell) ; Galvanometer (astatic) ; Resistance-Box ; Shunt. H. U. Adv., 88.

114 A. Find the resistance of a battery by Beetz' method (¶ 229). Read ¶¶ 226–228.

Apparatus : — 2 Batteries (2 Daniell, 1 Leclanché) ; Galvanometer (astatic) ; 2 Keys ; Resistance-Box.

H. U. Adv., 91.

ELECTROMOTIVE FORCE (Read § 139).

115 A. Find the electromotive force of a battery by the method of opposition (¶ 230 (7)). Use 5 or 6 Daniell cells and 3 Bunsen cells in series, with an astatic galvanometer and resistance-box. Estimate the electromotive force of the Daniell cells from that of the single cells tested in No. 112 A. (See ¶ 230 (2)). From this find that of the Bunsen cells. Read § 21 and § 145.

Apparatus: — Named above.

H. U. Adv. 93.

NOTE. If no number of Bunsen cells can be made to balance (approximately) any whole number of Daniell cells, notice the deflection of the galvanometer (which should be small) in two cases, and use the method of interpolation (§ 41).

116 A. Find the electromotive force of a Bunsen cell by Wiedemann's method (¶ 231).

Apparatus: — 2 Batteries (1 Bunsen, 2 Daniell); Galvanometer (S. R. or D. R.). H. U. Adv., 95.

117 A. Find corrections for a Volt Meter (¶ 231). Plot the results (Fig. 260). Read § 139.

Apparatus: — B. A. Bridge; Battery (2 Daniell); Galvanometer (astatic with extra slides); Resistance-Box.

H. U. Adv., 92.

118 A. Find the electromotive force of a Bunsen and a Leclanché cell by a volt-meter (¶ 235).

Apparatus: — Batteries (1 Bunsen, 1 Leclanché, &c.); Galvanometer (astatic); Resistance-Box.

H. U. Adv., 74.

119 A. Find the electromotive force of a Daniell cell by Poggendorff's absolute method (¶ 237).

Apparatus: — 2 Batteries (1 Daniell, 1 or 2 Bunsen); 2 Galvanometers (astatic and S. R. or D. R.); Resistance-Coil. H. U., Extra.

120 A. Find the efficiency of an electric motor (¶ 238, I).

Apparatus: — 2 Balances (spring); Battery (2 or 3 Bunsen); Clock; 2 Galvanometers (astatic and S. R. or D. R.); Motor (electric, small); Revolution Counter; Resistance-Box. H. U., *omit.*

Review Chap. X.

Review Chap. I–III.

General Review.

The list of experiments given above covers the ground of 42 of the Harvard elementary experiments, viz.: Nos. 1–4; 6–32; 34–42, and 44–45. It covers also the ground of 64 advanced experiments, viz.: Nos. 1–4; 7–18; 24–27; 29; 32; 35; 41–42; 45–46; 51–58; 60; 62–66; 68–71; 73–78; 81–82; 85–86; 89–90; 91–93; 95, and 98–99.

Two of the elementary experiments have practically been counted double, so that the real equivalent is 40 elementary experiments. To replace 11 of the advanced experiments anticipated by the elementary course, viz.: Nos. 17, 32, 41, 45, 46, 54, 56, 62, 63, 76, and 77, eleven extra experiments are suggested, namely, Nos. 9 A, 11 A, 23 A, 27 A, 33 A, 39 A, 59 A, 63 A, 68 A, 70 A, and 119 A. The exact correspondence of the regular experiments is shown in the schedule below. [The brackets indicate repetition.]

Elementary Course.

Harvard Elem. No.	First List No.	Harvard Elem. No.	First List No.	Harvard Elem. No.	First List No.
1	82 A	17	77 A	33	Omitted
2	81 A	18	74 A	34	49 A
3	78 A	19	72 A	35	53 A
4	79 A	20	73 A	36	50 A
5	Omitted	21	85 A	37	[52 A] & 54 A
6	28 A	22	37 A & 38 A	38	51 A
7	1 A & 2 A	23	31 A	39	[54 A]
8 }	{ 6 A, 7 A	24	35 A	40	88 A
9 }	{ & 4 A	25	29 A	41	95 A
10	3 A [21 A & 32 A]	26	30 A	42	96 A
11	10 A	27	42 A	43	Omitted
12	76 A	28	44 A	44	104 A
13	84 A	29	45 A	45	105 A & 111 A
14	75 A	30	69 A	46	Omitted
15	80 A [79 A]	31	62 A		
16	[79 A]	32	60 A		

Advanced Course.

Harvard Adv. No.	First List No.	Harvard Adv. No.	First List No.	Harvard Adv. No.	First List No.
1	5 A	35	55 A & 56 A	71	97 A & 102 A
2	24 A	41	[53 A]	73	98 A
3	25 A	42	52 A	74	118 A
4	26 A	45	[51 A]	75	112 A
7	12 A	46	[51 A]	76	[104 A]
8	13 A	50	57 A	77	[105 A & 111 A]
9	8 A	52	58 A	78	103 A
10	14 A	53	32 A	81	107 A
11	16 A	54	[81 A]	82	108 A
12	15 A	55	83 A & 19 A	85	106 A
13	20 A	56	[31 A]	86	99 A
14	22 A	57	36 A	88	113 A
15	21 A	58	34 A	89	110 A
16	17 A & 18 A	60	40 A	90	109 A
17	[72 A]	62	[42 A]	91	114 A
18	71 A	63	[44 A]	92	117 A
24	64 A	64	43 A	93	115 A
25	66 A	65	87 A	95	116 A
26	61 A	66	92 A	98	100 A & 101 A
27	67 A	68	90 A & 91 A	99	48 A
29	65 A	69	93 A		
32	[49 A]	70	94 A		

APPENDIX VII.

SECOND LIST OF EXPERIMENTS IN PHYSICAL MEASUREMENT INTENDED TO COVER THE GROUND REQUIRED FOR ADMISSION IN ELEMENTARY PHYSICS TO HARVARD COLLEGE.

NOTE. The experiments in this list are designated by the letter B. The abbreviations are the same as in the first list (see Appendix VI., page 1035).

1 B. Find the length, breadth, and thickness in *cm.* of a block of wood by several measurements of each of its dimensions (¶ 3). Read §§ 1, 2 and 5. Calculate the volume in *cu. cm.* by multiplying the length, breadth, and thickness together.

Apparatus: — Block (wooden solid), and a Gauge (vernier).
H. U. Elem., 7, I.

2 B. Find the weight in grams of the block used in 1 B., as in ¶ 2. Read §§ 6 and 9. Calculate as in ¶ 1 the density of the block.

Apparatus: — Balance (*b*); Block (wooden solid); Weights (*g*). H. U. Elem., 7, II.

3 B. Find the density of water, or better that of a saline solution of unknown strength, by loading a block of wood until it floats or sinks, indifferently (foot-note, page 2), then finding as in 1 B and 2 B the volume, weight, and density of the block. The latter is equal to the density sought. Read § 64.

Apparatus: — Balance; Block (hollow); Gauge (vernier); Weights (*g*); Lead shot and (salt) water.
H. U. Elem., 10, II.

4 B. Find the specific gravity of a block of wood by flotation in water. Mark the water-line in pencil at each corner, and calculate, as in 4 A, the specific gravity of the block. Read §§ 3 and 69.

Apparatus: — Block (wooden, solid); a Metre Rod; a pencil and water. H. U. Elem., 9, III.

5 B.* Find the weight required to sink a Nicholson's hydrometer to a given mark in water, at, below, and above the temperature of the room (¶¶ 6 and 7). Plot a curve as in Fig. 7, page 12. Read § 59.

Apparatus: — Brush (camel's-hair); Nicholson's Hydrometer; Thermometer and Weights (*cg*); Hot and cold water.

H. U. Elem., *omit.*

6 B.*. Find the weight in air of some steel bicycle balls, also that of a small wooden block, by Nicholson's Hydrometer (¶ 8).

Apparatus: — Balls (steel); Block, (small wooden); Brush (camel's-hair); Nicholson's Hydrometer; Thermometer and Weights (*cg*).

H. U. Elem., 8, I., 9 I.

7 B.* Find the weight in water of objects used in 6 B (¶ 10), and calculate their apparent specific gravity (§ 66).

Apparatus same as in 6 B.

H. U. Elem., 8 II., 9 II.

See Note under 7 A.

8 B. Find the (apparent) specific gravity of kerosene as follows: Weigh a bottle when empty, when filled with water, and when filled with kerosene. Calculate (by subtracting the weight of the empty bottle) the weights of water and of kerosene required to fill the bottle. Divide the weight of

* Experiments 5 B, 6 B, and 7 B, may be performed with a Jolly (spring) balance instead of Nicholson's Hydrometer.

kerosene by the weight of water to find the specific gravity in question.

Apparatus: — Balance (*b*); Specific Gravity Flask, kerosene and water. (More exact methods are considered in Exps. XI. and XIV.) H. U. Elem. 10, I.

9 B. Find the (apparent) specific gravity of kerosene by the 1st method of balancing columns (¶ 42, page 63). Read the 1st and last paragraphs of ¶ 43, also §§ 62 and 63. Use formula, page 66.

Apparatus: — Metre Rod and *U*-tube, with glass tubes and rubber couplings. H. U. Elem., 10, III.

10 B. Find the (apparent) specific gravity of glycerine by the 2d method of balancing columns (¶ 42, page 64). Readings and calculation the same as in 9 B.

Apparatus: — Metre Rod, Stop-cock and *Y*-tube, with glass tubes and rubber couplings. H. U. Elem., 10, IV.

11 B. Find the readings of a densimeter in glycerine, water, and kerosene, and plot curve of corrections as in Exp. XV. (¶¶ 39, 40, and 41).

Apparatus: — A Densimeter with jars containing glycerine, water, and kerosene.

H. U. Elem., *omit.*

12 B. Find the density of three saline solutions by means of a densimeter, applying corrections found in 11 B. (Exp. XV., ¶¶ 39, 40, and 41.)

Apparatus: — A Densimeter with 3 jars, containing different saline solutions.

H. U. Elem., *omit.*

13 B. Find roughly the density of air (as in Exp. XVI.) (¶¶ 44 and 45). Calculate the degree of exhaustion.

Apparatus: — Balance (*b*); Pump (Richards); Rubber Stopper (1 hole); Specific Gravity Flask; Stopcock; Thermometer; Weights (*g*).

H. U. Elem., 11.

14 B. Find the density of some coal-gas, as in Exp. XVIII. (¶ 46). Read §§ 70 and 81. See Tables 18, *d* and *e*.

Apparatus: — Balance (*b*); Rubber Stopper; Specific Gravity Flask; Thermometer; Weights (*b*) and coal gas.
H. U. Elem., Extra.

15 B. Find the temperature of the air (¶ 15), and the dew-point (¶ 16). Read ¶ 17. Obtain the relative humidity (Table 14 A), and the pressure of aqueous vapor (Table 15).

Apparatus: — Cup (nickel-plated), and Thermometer, with ice and salt. H. U. Elem., 22, II.

16 B. Find the maximum pressure of aqueous vapor at about 40° (¶ 81).

Apparatus: — Balance (*b*); Rubber Stopper; Specific Gravity Flask; Thermometer; Weights (*g*), and hot water.
H. U. Elem., 22, I.

17 B. Find the maximum pressure of ether vapor at about 20° by the second method suggested in ¶ 80.

Apparatus: — Medicine Dropper, Rubber Stopper (2 holes); Specific Gravity Flask; Thermometer, glass tubes, ether, and mercury. H. U. Elem., *omit.*

18 B. Find the barometer pressure as in the first paragraph of ¶ 13, testing as in the first paragraph of ¶ 14, then find the pressure of ether vapor by the first method suggested in ¶ 80. Read ¶ 80.

Apparatus: — Barometer (aneroid); Barometer Tube; Medicine Dropper; Thermometer, glass tubes, and mercury.
H. U. Elem., *omit.*

19 B. Find readings of a manometer under two or more different pressures (¶ 78). Read ¶ 77, and §§ 77, 78, and 79.

Apparatus: — Air Thermometer and Manometric Apparatus, with mercury. H. U. Elem., 6.

20 B. Find the mercurial pressure required to keep air in manometer from expanding when heated from 0° to 100° (¶ 76, as far as line 17, page 130). Read §§ 74, 75 and 76; also ¶ 76. Calculate *e* by formula, page 131.

Apparatus: — Air Thermometer; Manometric Apparatus; Steam Boiler; Steam Jacket; Thermometer.

H. U. Elem., 25.

21 B. Find the fixed points of an air-thermometer (first paragraph, ¶ 73). Read § 80 and ¶ 74. Calculate *e* by formula X., page 126.

Apparatus: — Air Thermometer; Steam Boiler; Steam Jacket; Thermometer. H. U. Elem., 26.

22 B. Find the fixed points of a mercurial thermometer (¶ 69), estimating tenths of a degree (see Fig. 52, ¶ 68). Read §§ 4 and 26; also first paragraph of ¶ 70. Refer to Table 14. Calculate corrections for the thermometer at 0° and 100°.

Apparatus: — Barometer (aneroid); Steam Boiler; Thermometer; and ice. H. U. Elem., 23.

23 B. Find the coefficient of linear expansion of a brass rod from about 20° to 100° (¶ 57). Read §§ 82 and 83.

Apparatus: — Brass Rod; Micrometer Frame; Steam Boiler; Steam Jacket; Thermometer. H. U. Elem., 24.

24 B. Find the specific heat of lead shot (¶ 94, I.). Read §§ 84, 85, 86 and 90. Use Formula VII., page 194.

Apparatus: — Balance (*b*); Bottle (ice water); Calorimeter; Thermometer; Weights (*g*), shot, ice, and water.

H. U. Elem., 27.

25 B. Find the latent heat of liquefaction of water, as in 44 A (First List of Experiments). Read ¶ 102, also §§ 87 and 91.

Apparatus: — Balance (*b*); Shot-heater; Stirrer; Thermometer; Weights (*g*), ice, and warm water.

H. U. Elem., 28.

26 B. Find the latent heat of vaporization of water essentially as in ¶ 103, but find the temperature of the water by a single observation *before* pouring it into the calorimeter, and cut off the steam when the water reaches the temperature of the room (see note under 44 A). Read § 88. Calculate the result by the formula of ¶ 104, *neglecting c.* Read ¶ 104.

Apparatus: — Balance (*b*); Steam Boiler; Steam Trap; Stirrer; Thermometer; Weights (*g*). H. U. Elem., 29.

27 B. Find the candle-power of a kerosene lamp by Bunsen's photometer (¶ 114, I.). Read § 94, ¶¶ 109 and 113. Reduce the candle-power of the lamp to 8 grams per hour. Use formula and reasoning of ¶ 113.

Apparatus: — Candle; Kerosene Lamp; Optical Bench; and Photometer. H. U. Elem., 34.

28 B. Find the relative intensities of the red, green, and violet rays reflected by a colored and by a white surface (¶ 246). Read ¶ 115.

Apparatus: — Colored Glasses; Kerosene Lamp; Optical Bench; and Colored Paper. H. U. Elem., *omit.*

29 B. Find the principal focal length of a lens by two different methods (¶ 116, (1) (2)). Read § 103.

Apparatus: — Chimney (perforated); Kerosene Lamp; Lens (magnifying); Optical Bench. H. U. Elem., 36.

30 B. Find the equivalent focal length of a compound lens, as in 51 A (First List of Experiments). Calculate the focal length (F) of the compound lens from that (F') of the lens in 29 B, by the formula (see 51 A) —

$$F = F' \times \frac{d}{d'}.$$

Read first two paragraphs of § 104. See Harvard List of advanced Physical Experiments, No. 45.

Apparatus: — Candle; Kerosene Lamp; 2 Lenses (doublet and magnifying); Metre Rod; Optical Bench.

H. U. Elem., 38.

31 B. Find several conjugate focal lengths of a lens (¶ 117, (1) (2), and, (3)). Note the size of the images (see § 104). Calculate the principal focal lengths of the lens. Use formula page 238.

Apparatus: — Chimney (perforated); Kerosene Lamp; Lens (magnifying); Metre Rod; Optical Bench.

H. U. Elem., 37, I.

32 B. Find the virtual foci of several (nearly) plane mirrors (¶ 118). Tell which are convex and which concave, remembering that the virtual images of *convex* mirrors are *nearer* than the objects producing them. Read § 104 and ¶ 118.

Apparatus: — Mirror (small), and Optical Bench.

H. U., Elem., 35.

33 B. Find 3 virtual foci of a long-focus converging lens (¶ 119, I.). Calculate the principal focal length.

Apparatus: — Lens (long-focus), and Optical Bench.

H. U. Elem., 37, II.

34 B. Find 3 virtual foci of a diverging lens (¶ 119, II.). Calculate the virtual principal focal length by the formula of ¶ 119.

Apparatus: — Lens (diverging), and Optical Bench.

H. U. Elem., *omit.*

35 B. Find the wave-length of sound from a tuning-fork in a rubber tube (131, I.). Read § 100.

Apparatus: — Metre Rod; Rubber Tube; Tuning-fork; *Y*-tube.

H. U. Elem., 32.

36 B. Find the wave-length of sound from a tuning-fork in a resonance tube (¶ 132). Read §§ 98 and 99. Notice that the lengths of the tube responding to a given fork are nearly proportional to the *odd* integers 1, 3, 5, &c.

Apparatus: — Resonance Tube, and Tuning-fork (A = 220).

H. U. Elem., Extra.

37 B. Find the pitch of a tuning-fork by the graphical method (¶ 139). Read §§ 7 and 96.

Apparatus: — Bow (violin); Clock; Smoked Glass Apparatus; Tuning-fork ($c = 64$). H. U. Elem., 31.

38 B. Find the velocity of sound (¶ 135 (1), (2), (3); ¶ 136, first paragraph, ¶ 137, III.). Read ¶ 138 and ¶ 135 (4), also §§ 8, 10, 92, and 93. Use formula II., page 281.

Apparatus: — Clock; Signalling Apparatus, and Tape Measure. H. U. Elem., 30.

39 B. Find the velocity of a bullet by a ballistic pendulum (¶ 147, (7)) as in 70 A. (First List of Experiments). Calculate the velocity V of the pendulum by the formula —

$$V = AB\sqrt{\frac{980}{AC}} \text{ (see § 109).}$$

and that (v) of the bullet by the formula —

$$v = \frac{(m + M)}{m} V \text{ (see ¶ 147 (7)).}$$

Read §§ 106 and 109.

Apparatus: — Clock, Metre Rod, and Pendulum (ballistic). H. U. Elem., *omit.*

40 B. Find the velocity acquired by a falling body (¶ 148). Read §§ 11, 107, and 108.

Apparatus: — Clock; Falling Bodies' Apparatus; Metre Rod. H. U. Elem., *omit.*

41 B. Find the length of a seconds, ½ seconds, and ¼ seconds pendulum (¶ 149). Tabulate results as on page 319. Read §§ 110, 111.

Apparatus: — Clock; Metre Rod; Pendulum (simple). H. U. Elem., 19.

42 B. Find the relative masses of two billiard balls suspended by cords as suggested on pages 312–313. See 73 A. (First List of Experiments).

Apparatus: — Balls (billiard); Cords; Metre Rod. H. U. Elem., 20.

43 B. Find the mass of a lead bullet by the method of oscillation (¶ 154). Read ¶ 155.

Apparatus: — Clock; Spiral Spring Apparatus; Weights (*cg*) and lead bullet.

H. U. Elem., 18.

44 B. Find corrections for a spring balance (¶ 158), and construct two tables (pages 339 and 340).

Apparatus: — Balances (spring 10 *k.*); Pulley; Weights (safety-valve). H. U. Elem., *omit.*

45 B. Find the weight in kilograms of a 28 lb. weight (¶ 159, 1); a 56 lb. weight (¶ 159, 2, and ¶ 159, 3); and a 4 lb. weight (¶ 159, 4), using a lever and one or two spring balances of 10 kilos capacity.

Apparatus: — Balances (spring, 10 *k.*); Lever; Weights (safety-valve), with cords. H. U. Elem., 14.

46 B. Find with 1 or 2 spring balances of 10 kilograms capacity and a system of cords, the weight in kilograms of a 4 lb. weight (¶ 159, 5) and of a 56 lb. weight (¶ 159, 6). Read § 105.

Apparatus: — Balances (spring, 10 *k.*); Weights (safety-valve), with cords. H. U. Elem., 12.

47 B. Find the weight of a board as in ¶¶ 160 and 161. Read § 112.

Apparatus: — Board (loaded); Pendulum (simple); Triangular supports; Weights (safety-valve); a pencil.

H. U. Elem., 17.

48 B. Find the stiffness of 5 beams by bending them (¶ 162). Read §§ 114 and 115.

Apparatus: — Beam (steel); Micrometer; Triangular support; Weights (*kg*). H. U. Elem., 3.

49 B. Find the (torsional) stiffness of two or more rods by twisting them (¶ 164). Read §§ 113 and 116.

Apparatus: — Balances (spring, 10 *k.*), and Torsion Apparatus. H. U. Elem., 4.

50 B. Find the (longitudinal) stiffness of a wire by stretching it. Measure the force (f), the amount of stretching (e), the length of wire (l), and its weight (w). Take the density (d) from Table 9. Calculate q by formula II., page 361; then calculate Young's modulus as explained in ¶ 166.

Apparatus: — Balances (spring, 10 *k.*); Metre Rod; fine steel wire. H. U. Elem., 2.

51 B. Find the breaking strength of several wires (first paragraph, ¶ 168). Weigh a known length of the wire, and calculate the length which would break under its own weight. Read ¶ 168.

Apparatus: — Balance (spring, 10 *k.*); Bobbins and wires. H. U. Elem., 1.

52 B. Find by two methods the coefficient of friction of wood on wood (¶ 171, I. and II.). Review § 105.

Apparatus: — Balance (spring, 10 *k.*); Board and plank; Weights (*kg*).

H. U. Elem., 13.

53 B. Find the efficiency of a pulley (1) for raising heavy weights, and (2) for multiplying motion (¶ 173). Read § 117.

Apparatus: — Balance (spring, 10 *k.*); Metre Rod; Tackle; Weights (safety-valve). H. U. Elem., 21.

54 B. Find the poles of a magnet by means of iron filings, and confirm by a small compass needle (¶ 179). Read §§ 126 and 127.

Apparatus: — Compass (vibrating); Magnet (compound) Iron filings; Photographic paper and pencil.

H. U. Elem., 40.

55 B. Find the magnetic dip by a dipping-needle (¶ 190). Read § 128.

Apparatus: — Dipping needle. H. U. Elem., Extra.

56 B. Find the relative strength of battery currents from a 1-fluid cell under given conditions (¶ 208, (1), to (8)). Read §§ 123, 124, 130, and ¶ 207. Reduce results as in ¶ 209.

Apparatus: — Battery (1 Daniell); Compass (vibrating); Galvanometer. (The porous cup is to be removed from the Daniell cell.) H. U. Elem., 41.

57 B. Find the deflection of a tangent compass at the centre of a coil of wire due to currents from a Daniell cell under the conditions of ¶ 208, (9) to (12). Weigh the zinc and the copper before and after the experiment. Read §§ 143, 144. Review ¶ 209.

Apparatus: — Balance (*b*); Battery (1 Daniell); Compass (surveying); 6 Galvanometers; Weights (*g*).

H. U. Elem. 42.

58 B. Find the length of copper wire about ¼ *mm.* in diameter (No. 31 B. W. G.), which can he substituted for a 1-ohm coil (*C*) in the circuit of a Daniell cell (*B*) and galvanometer (*G*), — see Fig. 243, page 476, — without changing the deflection. Repeat with a double wire, with a German silver wire of the same diameter, and with one of twice the diameter or 4 times the cross-section (about No. 25 B. W. G.). Read ¶ 218, also § 140.

Apparatus: — Battery (1 Daniell); Compass (surveying); Galvanometer; Resistance-Coil (1 ohm); and wires as stated.*

H. U. Elem., 44.

59 B. Find the (external) resistance of a circuit as follows: First, note the deflection of a galvanometer due to each one of two equal cells, then join the cells in series (Fig. 20, § 146), and include German silver wire enough in the circuit to give the same (average) deflection as before. Read § 138 and § 146. Calculate the resistance of this wire. This is equal to the value sought. For proof, see 105 A.

Apparatus: — Battery (2 Daniell); Compass (surveying); Galvanometer, with German silver wires.*

H. U. Elem., 45 I.

60 B. Find the resistance of a Daniell cell as follows: Note the deflection of each of two cells as in 59 B, and join them in multiple arc (Fig. 19, § 146). Include in the circuit enough German silver wire to give the same average deflection as before. Calculate the resistance of this wire, and multiply it by 2 to find the resistance sought. For Proof, see 111 A.

Apparatus: — Battery (2 Daniell); Compass (surveying); Galvanometer, with clamps and German silver wire.*

H. U. Elem., 45 II.

* It is not necessary to cut the wires in 58 B, 59 B, and 60 B. A greater or less length may be included between two clamps, as in ¶ 237. The wire should be kept straight, as in Fig. 249, page 486.

REVIEW.

Chapter I. (General Definitions), first 11 sections.

Chapter V. (Hydrostatics), omitting §§ 67, 68, 71, 72, and 73.

Chapter VI. (Heat), omitting § 89 on cooling.

Chapter VII. (Sound and Light), §§ 92, 93, 94, 96, 98, 99, 100, 103, and 104.

Chapter VIII. (Force and Work), as far as § 118.

Chapter IX. (Electricity and Magnetism), §§ 123, 124, 126, 127, 128, 130.

Chapter X. (Electromotive Force, and Resistance), §§ 138, 140, 143, 144, 146.

The Second List of Experiments is intended to cover the ground of 40 Exercises in Elementary Physics required for admission to Harvard College, *viz.* Nos. 1–4; 6–14; 17–32; 34–38; 40–42; and 44–45. In most cases the correspond-

ence is exact; other cases are designated in the table below by an asterisk (*). The course of reading recommended covers the principles of at least three additional exercises, and three extra experiments are suggested. The ground covered for examination is therefore about equivalent to the 46 exercises of the Harvard elementary pamphlet. The laboratory work is divided into 50 experiments (assuming that 10 of the 60 are omitted as indicated). As these experiments all involve measurements, they are on the average fully as difficult as those recommended by the Harvard pamphlet. This course would be offered only by students who are ambitious to learn more about *physical measurement* than is thought desirable to require of all candidates for admission to Harvard College in elementary physics.

The exact correspondence of the second list of experiments with the "Descriptive List of Elementary Physical Experiments" published by Harvard University, October, 1889, is shown by the table below.

Harvard Elem., No.	Second List, No.	Harvard Elem., No.	Second List, No.	Harvard Elem., No.	Second List, No.
1	51 B.	14	45 B.	32	35 B.
2	50 B.	15	[§ 113]	33	Omit
3	48 B.	16	[¶ 164]	Extra	36 B.
4	49 B.	17	47 B.	34	27 B.
5	[§§ 62–63]	18	43 B.	35	32 B.
6	19 B.	19	41 B.	36	29 B.
7 I.	1 B.	20	42 B.	37 I.	31 B.
7 II.	2 B.	21*	53 B.*	37 II.	33 B.
8 I. & II.	6 B.	22 I. *	16 B.*	38	30 B.
9 I. & II.	7 B.	22 II.	15 B.	39	Omit
9 III.	4 B.	23	22 B.	40	54 B.
10 I.	8 B.	24	23 B.	Extra	55 B.
10 II.	3 B.	25	20 B.	41	56 B.
10 III.	9 B.	26	21 B.	42	57 B.
10 IV.	10 B.	27	24 B.	43	Omit
11	13 B.	28	25 B.	44	58 B.
Extra	14 B.	29	26 B.	45*	59 B. &
12	46 B.	30	38 B.		60 B.*
13	52 B.	31	37 B.	46	Omit

* Cases of only approximate correspondence (3 such cases in all).

APPENDIX VIII.

ADVANCED PHYSICS.

THIRD LIST OF EXPERIMENTS IN PHYSICAL MEASUREMENT INTENDED TO COVER THE GROUND REQUIRED FOR ADMISSION TO HARVARD COLLEGE IN ADVANCED PHYSICS.

NOTE. The experiments in this list are designated by the letter C. The abbreviations are the same as in the first list (Appendix VI., page 1035). Before beginning the experiments the student should review those sections in Part III., already mentioned (see Appendix VII., page 1077), and should read in addition Chapters II. and IV., omitting §§ 51, 52, and 61; also §§ 48 and 49 of Chapter III.

1 C. Find the sensitiveness of a balance with loads of 0, 20, 50, and 100 grams in each pan, (¶¶ 20, 21). Plot the results (Fig. 16). Read ¶ 22. Review §§ 26, 30, 59.

Apparatus: — Balance (*a*); Weights (*cg*).

H. U. Adv., 9.

2 C. Find the ratio of the arms of a balance (¶ 23). Repeat two or three times. Reduce as in ¶ 24. Read § 46. Estimate probable error (§ 50).

Apparatus: — Balance (*a*); Weights (*cg*).

H. U., Extra.

3 C. Find a correction for the reading of a barodeik (¶ 18), by means of a hygrodeik (¶ 15) and an aneroid barometer. Use Tables 19, 20. Read § 71.

Apparatus: — Balance (*a*); Barodeik; Barometer (aneroid); Hygrodeik; Thermometer; Weights (*cg*.).

H. U. Adv., 7.

4 C. Find the weight of a glass ball in air by a double weighing (¶ 28). Weigh also a piece of cork coated with varnish. Read §§ 35, 44, 67, and 72. Reduce the results to vacuo.

Apparatus: — Balance (*a*); Ball (glass); Rings (small); Weights (*cg*). H. U. Adv., 8.

5 C. Find the weight of a glass ball in water (¶ 29). Review §§ 64, 65, 66, and 67. Read § 68. Calculate the volume and density of the ball.

Apparatus: — Arch (hydrostatic); Balance (*a*); Ball (glass); Beaker; Brush (camel's-hair); Stirrer; Thermometer; Weights (*cg*). Supplies: Wire and water.

H. U. Adv., 10.

6 C. Find the weight of the cork (in No. 4 C) in water by attaching a sinker to it, and weighing the sinker in water with and without the cork (¶ 29). Calculate the density of the cork. Review § 34. Consider what assumptions you have made in this and in other experiments with the hydrostatic balance. Test the accuracy of one or more of these assumptions by reweighing the cork in air *after* weighing it in water.

Apparatus: — Arch (hydrostatic); Balance (*a*); Beaker; Brush (camel's-hair); Cork; Sinker; Weights (*cg*). Supplies: Wire and water. H. U. Adv., 12.

7 C. Find the weight of a glass ball (of No. 5 C) in alcohol at an observed temperature (¶ 30). Calculate the density of the alcohol (¶ 31).

Apparatus: — Arch (hydrostatic); Balance (*a*); Ball (glass); Beaker; Brush (camel's-hair); Stirrer; Thermometer; Weights (*cg*). Supplies: Wire and alcohol.

H. U. Adv., 11.

8 C. Find the capacity of a Specific Gravity Bottle (¶ 32). Read ¶ 33.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*); Water.

H. U. Adv., 13.

9 C. Find the density of alcohol by the Specific Gravity Bottle, and calculate the strength of the alcohol (¶ 38). Use Table 27.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*); Alcohol.

H. U. Adv., 15.

10 C. Find the volume of some steel balls by the Specific Gravity Bottle (¶ 34). Read ¶ 35; also § 38. Calculate the density of the balls.

Apparatus: Balance (*a*); Balls (steel); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*); Water.

H. U. Adv., 14.

11 C. Find the volume of some crystals of sulphate of copper by the use of alcohol (¶¶ 36, 37), and calculate their density.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*). Supplies: Alcohol and crystallized sulphate of copper.

H. U., *omit.*

12 C. Find the correction for one reading of a vernier gauge (¶ 50 I.). Read ¶ 47, but use Table 3 H. Read ¶¶ 48 and 49, also §§ 37, 43, and 73.

Apparatus: — Ball (glass); Gauge (vernier); Lens (magnifying).

H. U. Adv., 2.

13 C. Find the pitch of a screw (¶ 50, II.).

Apparatus: — Balls (steel); Micrometer gauge.

H. U. Adv., 3.

14 C. Find the constants of a spherometer (¶¶ 51 and 54).

Apparatus: — Ball (glass); Plate Glass; Spherometer.

H. U. Adv., 4.

15 C. Find the radii of curvature of 2 spherical surfaces (¶ 55). Read ¶ 56.

Apparatus: — Lens (magnifying); Spherometer.

H. U., Extra.

16 C. Find the capacity of a capillary tube by means of mercury. See ¶ 169, II., and ¶ 170. Read § 39.

Apparatus: — Balance (*a*); Capillary Tube; Weights (*cg*); Mercury. H. U. Adv., 55, II.

17 C. Find the fixed, middle, and quarter points of a mercurial thermometer (¶¶ 66, 67, 68, 69, and 70). Read § 36, (3).

Apparatus: — Beaker (for ice); Bunsen Burner; Steam Boiler; Thermometer. Supplies: Gas, ice and water (or steam). H. U. Adv., 56.

18 C. Find the coefficient of expansion of water between about 20° and 100° (¶ 59). Read ¶¶ 60 and 61. Review §§ 62 and 63.

Apparatus: — Expansion Apparatus with accessories, supply of water and steam. H. U. Adv., 53.

19 C. Find the coefficient of expansion of alcohol from about 20° to 40° or 50° by the Specific Gravity Bottle (¶¶ 62, 63). Review § 82.

Apparatus: — Balance (*a*); Specific Gravity Bottle; Stirrer; Thermometer; Weights (*cg*). Supplies: Alcohol and hot water. H. U., Extra.

20 C. Find the coefficient of expansion of glass by the weight thermometer (¶ 240). Review § 83.

Apparatus: — Balance (*a*); Bunsen Burner; Steam Boiler; Steam Jacket; Thermometer (weight); Weights (*cg*). Supplies: Gas, ice, mercury, and water (or steam).

H. U. Adv., 58.

21 C. Find the boiling point of one or more liquids, and the melting point of paraffine (¶¶ 83, 84).

Apparatus: — Stopper (1 hole); Test-tube; Thermometer. Supplies: Hot water, paraffine, alcohol, etc.

H. U. Adv., 57.

CALORIMETRY (Review §§ 84–91).

22 C. Find the rate of cooling of a calorimeter (¶¶ 85, 87). Read ¶ 86, also §§ 47, 89.

Apparatus: — Calorimeter; Clock; Stirrer; Thermometer. Supply of hot water. H. U., Extra.

23 C. Find the thermal capacity of a calorimeter with thermometer and stirrer (¶ 90 (1) I; ¶ 91, I). Read §§ 16. 45; Review § 85.

Apparatus: — Balance (*b*); Calorimeter; Clock; Stirrer; Thermometer; Weights (*g*). Supply of hot water.

H. U. Adv., 60.

24 C. Find the thermal capacity of a thermometer of a stirrer, and of a calorimeter, as in (¶ 90, 2). Use formula III., ¶ 91.

Apparatus: — A Balance (*b*); Calorimeter; Measuring glass; Stirrer; Thermometer and water.

H. U. Adv., 61.

25 C. Find the specific heat of turpentine by the method of mixture (¶ 96, I.). Read ¶ 95. Use formula VIII., ¶ 98. Review § 90.

Apparatus: — Balance (*b*); Calorimeter; Stirrer; Thermometer; Weights (*g*). Supplies: Turpentine cooled to 0°, and hot water.

NOTE. Students who have not already determined the specific heat of lead shot should substitute this determination (¶ 94, I.). H. U. Adv., 62.

26 C. Find the specific heat of alcohol by the use of lead shot (¶ 96, II.).

Apparatus: — Balance (*b*); Bunsen Burner; Calorimeter; Steam Shot-heater; Thermometer; Weights (*g*). Supplies: Gas, alcohol (at 0°), water (or steam) and lead shot.
H. U., *omit.*

27 C. Find the specific heat of alcohol or turpentine by an electrical method in 43 A. (First List of Experiments, see Appendix VI.)

Apparatus: — Balance (*b*); Battery (2 Bunsens); 2 Calorimeters; 2 Resistance-coils; 2 Stirrers; 2 Thermometers; Weights (*g*). Supplies: Alcohol and water, connecting wires.
H. U. Adv., 64.

Review Exp. 25 B (H. U. Elem., 28 = H. U. Adv., 63).

28 C. Find the heat of combination of zinc and nitric acid (¶ 105, I., and ¶ 106).

Apparatus: — Balance (*a*); Calorimeter with glass lining; Clock; Stirrer; Thermometer; Weights (*cg*). Supplies: Zinc filings and dilute nitric acid.
H. U., *omit.*

29 C. Find the heat of combination of zinc oxide and nitric acid (¶ 105, II., and ¶ 106).

Apparatus: — Balance (*a*); Calorimeter with glass lining; Clock; Stirrer; Thermometer; Weights (*cg*). Supplies: Zinc oxide and dilute nitric acid.
H. U., *omit.*

RADIANT HEAT (Review §§ 93, 94; Read § 95).

30 C. Find the candle-heat-power of a kerosene lamp (¶ 111), and calculate that of a lamp burning 8 grams of kerosene per hour (¶ 113).

Apparatus: — Balances (*b*); Candle; Clock; Galvanometer (astatic); Kerosene Lamp; Optical Bench; Thermopile; Weights (*g*).
H. U. Adv., 99.

Review Exp. 27 B (H. U. Elem., 34 = H. U. Adv., 32).

LIGHT.

Review Exps. 30 B, 31 B, 32 B, 33 B (H. U. Elem., 35–37 = H. U. Adv., 41, 42, 43, 45).

31 C. Find the zero-reading of a sextant (¶ 123). Read §§ 31, 32, 97.

Apparatus: — A Sextant. H. U. Adv., 35 I.

32 C. Find by a sextant the angular semidiameter of the sun (¶ 124, I.).

Apparatus: — A Sextant. H. U. Adv., 35, II.

33 C. Find by a sextant the distance of a terrestrial object of known magnitude (¶ 124, II., and ¶ 136).

Apparatus: — A Sextant. H. U., *omit.*

34 C. Find the latitude and longitude of a place (¶¶ 242, 243, Tables 44 A–44 G).

Apparatus: — An Artificial Horizon and a Sextant.

H. U., *omit.*

35 C. Find by a sextant the three angles of a prism (¶ 125, I.).

Apparatus: — A small Prism and a Sextant.

H. U. Adv., 51.

36 C. Find by a spectrometer the three angles of a prism (¶ 126).

Apparatus: — A small Prism; a (kerosene) Lamp; a Spectrometer. H. U. Adv., 50.

37 C. Find the angle of minimum deviation for a ray of sodium light passing through a prism angle of known magnitude (¶¶ 126, 127). Read ¶ 128, and § 102.

Apparatus: — Prism (used in No. 36 C); Sodium flame (with slit); Spectrometer (or sextant).

H. U. Adv., 52.

38 C. Find the distance between the lines of a diffraction grating (¶ 130). Review § 100. Read § 101 and ¶ 129.

Apparatus: — Diffraction Grating: Sodium flame (with slit); Spectrometer (or sextant). H. U., Extra.

SOUND (Read §§ 92 and 96).

Review Exps. 36 B and 37 B (H. U. Elem., 28 and 31 = H. U. Adv., 26 and 22).

39 C. Find the pitch of a tuning-fork by the toothed wheel (¶ 144). Read ¶ 145.

Apparatus: — A Toothed Wheel Apparatus, and a Tuning-fork (C = 64). H. U., Extra.

40 C. Find the musical interval between two tuning-forks by means of a monochord (¶ 133 III). Read ¶ 134.

Apparatus: — A Monochord and 2 Tuning-forks (A = 216 to 220, C = 256). See note under 64 A, First List of Experiments, Appendix VI. H. U. Adv., 24.

41 C. Find by Lissajous' curves (¶ 143) the musical interval between 2 C-forks 2 "octaves" apart; also find the musical interval between the higher of these forks and a G♯ fork, two "octaves" and a "third" below it. Read ¶¶ 134 and 142.

Apparatus: — Lens (small); 3 Tuning-forks (C = 256, C = 64, G♯ = 51.2). (Kerosene lamp for smoking, and sealing wax.)

See Note under 65 A, First List of Experiments, Appendix VI. H. U. Adv., 29.

42 C. Find the pitch of a set of forks, covering a known musical interval, by the method of beats (¶ 141). Read ¶ 140.

Apparatus: — A Clock and 5 Tuning-forks; G♯ = 51.2, A = 54, A♯ = 57, B = 60, C = 64. H. U. Adv., 25.

See Note under 66 A, First List of Experiments, Appendix VI.

43 C. Find the pitch of the note due to longitudinal vibration in a wire (¶ 248; I.), either by a pitch-pipe (Fig. 273), or (in the absence of a musical ear) by a resonance tube

(¶ 132, and ¶ 134, II.). Calculate the velocity of sound in the wire (¶ 248).

Apparatus: — A Pitch-pipe (or Resonance Tube); Tape measure; Wires; Cloth, resin, etc. H. U. Adv., 27.

44 C. Find the pitch of the note due to torsional vibrations in a wire (¶ 248, II.), either by a pitch-pipe or by a resonance tube. Calculate the velocity of these torsional vibrations in the wire.

Apparatus: — A Pitch-pipe (or Resonance Tube); Tape measure; Wires; Cloth, resin, etc. H. U., Extra.

DYNAMICS (Read §§ 28, 29, 111, ¶ 138).

45 C. Find the length and time of oscillation of an irrotational pendulum (¶ 151, II.). Read ¶ 150 and ¶ 152, §§ 40 and 61. Obtain g from table, ¶ 153.

Apparatus: — Clock; Gauge (vernier); Metre Rod; Pendulum (irrotational). H. U. Adv., 17.

46 C. Find the coefficient of torsion of a wire by a torsion balance (¶ 165). Read § 12; review §§ 13 and 116.

Apparatus: — Gauge (micrometer); Metre Rod; Torsion Balance; Torsion Head; Weights (*cg*). H. U., Extra.

47 C. Find Young's modulus of elasticity for a wire (¶ 167). Review § 114.

Apparatus: — Gauge (micrometer); Micrometer (electric) Tape measure; Weights (*kg*); Young's Modulus Apparatus. H. U. Adv., 54.

48 C. Find the surface tension of water by means of the capillary tube of No. 16 C (¶ 169, II.). Read ¶ 170.

Apparatus: — Beaker; Capillary Tube; Metre Rod; Thermometer. H. U. Adv., 55 I.

ENERGY (Read §§ 14, 15, 117–121).

49 C. Find the coefficient of hydraulic "resistance" for a rubber tube (¶ 172, page 378). Calculate the coefficient of friction for water.

Apparatus: — Balance (rough); Blocks; Clock; 2 Jars; Weights (*kg*). H. U., *omit*.

50 C. Find the efficiency of a water motor (¶ 174). Read ¶ 175. Read §§ 14, 15, 117, 118.

Apparatus: — Balance (rough); Clock; 2 Spring Balances; Jar; Tape measure; Water Motor (with pressure gauge); Weights (*kg*). H. U., *omit*.

51 C. Find (roughly) the mechanical equivalent of heat by means of lead shot (¶ 177, first paragraph). Read ¶ 176 and ¶ 178.

Apparatus: — Pasteboard Tube (with corks); a Thermometer and some Lead Shot. H. U. Adv., 65.

MAGNETISM.

52 C. Find the attraction and repulsion between two parallel magnets at a given distance (¶ 180). Estimate the strength of the poles (¶ 181). Read §§ 17, 129.

Apparatus: — Balance (*a*); Blocks (*cu. cm*); Gauge (vernier); 3 Magnets (compound); Weights (*cg*).

H. U., Extra.

NOTE. In this and in following experiments, the distance between the poles of the (short) compound magnets may be called equal to $\frac{7}{10}$ the length of the magnet. See ¶ 179.

53 C. Find the couple exerted by the earth's magnetism upon 3 magnets by means of torsion (¶ 182). Estimate "H."

Apparatus: — 3 Magnets (compound); Torsion Head and Wire tested in No. 46 C.; Wax and pins to serve as sights.

H. U. Adv., 67.

54 C. Find the deflection of a compass needle due to magnets of known strength (from No. 52 C.) at a given distance (¶ 183). Read ¶¶ 184, 185. Estimate "H." Calculate the true value of "H" from the estimates in Nos. 53 C. and 54 C.

Apparatus: — Compass (surveying); Magnets (compound); Metre Rod. H. U. Adv., 68.

55 C. Find the distribution of magnetism on a magnet by the method of vibrations (¶ 186). Plot a curve (Fig. 205). Estimate the distance between the poles.

Apparatus: — Clock; Magnet (vibrating needle); Magnet (long-bar); Metre Rod; Test Tube. H. U. Adv., 66.

56 C. Find the distribution of magnetism on a magnet by means of an induction coil (¶ 189). Plot the curve and estimate the distance between the poles as in No. 55 C. Read ¶¶ 187 and 188.

Apparatus: — Galvanometer (astatic); Helix (sliding); Magnet (long-bar); Metre Rod. H. U. Adv., 69.

57 C. Find the magnetic dip by the earth-inductor (¶ 192). Read ¶¶ 190, 191, and § 147.

Apparatus: — Earth-Inductor; Galvanometer (astatic, loaded so as to answer for a ballistic galvanometer), and a Level. H. U. Adv., 70.

ELECTRICAL CURRENT MEASURE (Read §§ 18, 19, 130–133).

58 C. Find the constant and reduction factor of a single ring tangent galvanometer (¶¶ 198 and 199, formulæ (5) and (6)).

Apparatus: — A Galvanometer (S. R.), a Gauge (long vernier), and a Tape Measure. H. U. Adv., 71, I.

59 C. Find the reduction factor of a double-ring galvanometer by the method of comparison (¶ 201). Read ¶ 200.

Apparatus: — Battery (2 Daniell); 2 Commutators; 2 Galvanometers (S. R. and D. R.), and connecting wire.

H. U. Adv., 73.

60 C. Find the reduction factor of an ammeter by the method of comparison (¶ 210).

Apparatus: — An Ammeter; Battery (2 or 3 Bunsen); 2 Tangent Galvanometers (S. R. and D. R.).

H. U. Adv., *omit.*

61 C. Find the reduction factor of an astatic galvanometer with shunt, by the method of comparison (¶ 201), as in 99 A (First List of Experiments, Appendix VI.). Note what plugs are removed from the rheostat, also the length, diameter, and material of the shunt. If the resistances R, G, and S of the rheostat, galvanometer, and shunt are known, calculate the reduction factor of the galvanometer without the shunt from that of the combination (I), by the formula

$$i = I \times \frac{S}{R + G + S}.$$

Apparatus: — A Battery (1 Daniell); 2 Galvanometers (astatic and D. R.); a Gauge (micrometer); a Metre Rod; a Resistance-box; 1 metre of German silver wire (about No. 25 B. W. G.). H. U. Adv., 86.

62 C. Find the reduction factor of a dynamometer by comparison with a single-ring galvanometer (¶ 204). Read ¶ 202. Let C be the current indicated by the galvanometer, and a the angle of torsion in the dynamometer; then the reduction factor (D) is

$$D = C \div \sqrt{a}.$$

Apparatus: — A Battery (3 Bunsen or 6 Daniell); 2 Commutators; a Dynamometer; a Galvanometer (S. R.), and connecting wires. H. U. Adv., 98, I.

63 C. Find by measurement the reduction factor of a dynamometer (¶ 203). Read §§ 134, 135. Review § 116. Use the formula

$$D = 10\sqrt{\frac{t}{KA}}.$$

Calculate the current C in No. 63 C. by the formula

$$C = D\sqrt{a}$$

then find I and H as in ¶ 204.

Apparatus: — A Dynamometer; a Gauge (vernier long); [a Torsion Balance, and Weights (cg)].

H. U. Adv., 98, II.

64 C. Find the reduction factor of a galvanometer by the electro-chemical method (¶ 205). Calculate "H" (¶ 206). Read § 142. Review §§ 143, 144.

Apparatus: — Balance (a); Battery (1 Daniell); Clock; Commutator; Galvanometer (S. R.); Weights (cg), and a spiral of copper wire. H. U. Adv., 71, II.

ELECTRICAL RESISTANCE (Read §§ 20, 136, 137).

65 C. Find the electrical resistance of a coil of wire by the method of heating (¶¶ 212, 213).

Apparatus: — Balance (b); Battery (2 Bunsen); Calorimeter; Resistance-coil; Stirrer; Thermometer; Weights (g).

H. U. Adv., 78.

Review Exp. 58 B (Elem. 44 = Adv. 76).

66 C. Find the electrical resistance of a conductor by means of a differential galvanometer (¶ 216).

Apparatus: — Battery (1 Daniell); a Galvanometer (astatic with differential connections); the Helix of No. 56 C.; a Key; and a Resistance-box. H. U. Adv., 85.

67 C. Find gross errors (if any) in a resistance-box by means of a Wheatstone's Bridge (¶ 217). Use as a (rough) standard of comparison the resistance-coil tested in No. 65 C. Read §§ 42 and 141. Review § 45.

Apparatus: — B. A. Bridge; Battery (1 Daniell); Galvanometer (astatic); Resistance-box and Resistance-coil.

H. U. Adv., 81.

68 C. Find by Wheatstone's Bridge the resistance of the shunt used in No. 61 C, and calculate the specific resistance of the material of which it is made (¶ 219). Read ¶ 217.

Apparatus: — B. A. Bridge; Battery (1 Daniell); Galvanometer (astatic), and Shunt. H. U. Adv., 82.

69 C. Find the resistance of the galvanometer used in 61 C by Thomson's method (¶ 220). Read ¶ 221.

Apparatus: — B. A. Bridge; Battery (1 Daniell, shunted); Galvanometer (astatic); Key; Magnet (small compound); Resistance-box. H. U. Adv., 90.

70 C. Find the resistance of a battery by Mance's method (¶ 222). Read ¶ 222 *a*.

Apparatus: — B. A. Bridge; a Battery (1 Daniell); a Galvanometer (astatic); a Key; a Magnet (compound, small); and a Resistance-box. H. U. Adv., 89.

71 C. Find the resistance of a tangent galvanometer by the use of a shunt (¶ 223, I.). Read ¶ 224, I.

Apparatus: — Battery (1 Daniell); 2 Galvanometers (S. R. and D. R.); Resistance-box (or shunt).

H. U., Extra.

Review Experiment 60 B (Elem. 45 = Adv. 77).

72 C. Find the resistance of a battery by Ohm's method (¶ 225). Review § 138.

Apparatus: — A Battery (1 Daniell); a Galvanometer (S. R.), and a Resistance-box. H. U. Adv., 75.

NOTE. The battery cell should be marked so that it can be identified later on.

73 C. Find the resistance of a battery by Thomson's method, as in 113 A (First List of Experiments, Appendix VI.).

Apparatus: — Battery (1 Daniell); Galvanometer (astatic); Resistance-box, and shunt. H. U. Adv., 88.

74 C. Find the resistance of a battery by Beetz' method (¶ 229). Read ¶¶ 226–228.

Apparatus: — 2 Batteries (2 Daniell, 1 Leclanché); Galvanometer (astatic); 2 Keys; Resistance-box. H. U. Adv., 91.

ELECTROMOTIVE FORCE (Read §§ 21 and 139; Review §§ 137, 138, 145).

75 C. Find the electromotive force of a battery by the method of opposition (¶ 230 (7)). Use 5 or 6 Daniell cells and 3 Bunsen cells in series, with an astatic galvanometer and resistance box. Estimate the electromotive force of the Daniell cells from that of the single cell tested in No. 72 C. (see ¶ 230 (2)). From this find that of the Bunsen cells. Read § 41.

Apparatus: — Named above. H. U. Adv., 93.

NOTE. See note under 115 A (First List of Experiments, Appendix VI.).

76 C. Find the electromotive force of a Bunsen cell by Wiedemann's method (¶ 231).

Apparatus: — 2 Batteries (1 Bunsen, 2 Daniell); a Galvanometer (S. R. or D. R.). H. U. Adv., 95.

77 C. Find corrections for a volt-meter (¶ 234). Plot the results (Fig. 260).

Apparatus: — B. A. Bridge; Battery (2 Daniell); Galvanometer (astatic with extra slider); and a Resistance-box. H. U. Adv., 92.

78 C. Find the electromotive forces of a Bunsen and a Leclanché cell by a volt meter (¶ 235).

Apparatus: — Batteries (1 Bunsen, 1 Leclanché, &c.); a Galvanometer (astatic), and Resistance-box.

H. U. Adv., 94.

79 C. Find the electromotive force of a Daniell cell by Poggendorff's absolute method (¶ 237).

Apparatus: — 2 Batteries (1 Daniell, 1 or 2 Bunsen); 2 Galvanometers (astatic and S. R. or D. R.); Resistance-coil (in calorimeter). H. U. Adv., 96.

80 C. Find the efficiency of an electric motor (¶ 238).

Apparatus: — 2 Balances (spring); Battery (2 or 3 Bunsen); Clock; 2 Galvanometers (astatic and S. R. or D. R.); Motor (electric, small); Revolution Counter; Resistance-box. H. U., *omit.*

The "third list" of experiments given above contains 60 regular and 10 "extra" experiments. The latter are intended to take the place of 10 advanced experiments, the principles of which have probably been anticipated in an elementary course. The corresponding experiments in the elementary course are marked for review, with references to the Harvard elementary pamphlet, and to the "second list" (B, Appendix VII.), where they may be found. The student will do well in any case to prepare himself for examination upon the principles of these 10 elementary experiments, which together with the 60 regular experiments of the "third list" are thought to cover the ground of 66 of the 100 experiments in Physical Measurement published by Harvard University, June, 1890.

The 66 experiments have been selected as follows: —

13 in Mechanics and Hydrostatics; Nos. 2–4, 7–15, and 17.

6 in Sound; Nos. 22, 24–27, and 29.

9 in Light; Nos. 32, 35, 41–43, 45, and 50–52.

19 in Heat; Nos. 53–58; 60–71, and 73.

19 in Magnetism and Electricity; Nos. 75–78, 81–82, 85–86, 88–96, and 98–99.

The exact correspondence between these 66 experiments in "advanced physics" and those contained in the "third list" designated by the letter C is shown by the table below. The experiments marked B. are those taken from the "second list" (Appendix VII.). It is understood that these experiments are to be offered for admission to Harvard College either in elementary or in advanced physics. In the former case, they should be replaced by an equal number of experiments marked "extra" in the "third list," in order to meet the college requirements for admission in Advanced Physics.

Harvard Adv., No.	Third List, No.	Harvard Adv., No.	Third List, No.	Harvard Adv., No.	Third List, No.
2	12 C	43	33 B	71 II.	64 C
3	13 C	45	30 B	73	59 C
4	14 C	50	36 C	75	72 C
7	3 C	51	35 C	76	58 B
8	4 C	52	37 C	77	60 B
9	1 C	53	18 C	78	65 C
10	5 C	54	47 C	81	67 C
11	7 C	55 I.	48 C	82	68 C
12	6 C	55 II.	16 C	85	66 C
13	8 C	56	17 C	86	61 C*
14	10 C	57	21 C	88	73 C
15	9 C	58	20 C	89	70 C
17	45 C	60	23 C	90	69 C
22	37 B	61	24 C	91	74 C
24	40 C	62	25 C	92	77 C*
25	42 C	63	25 B	93	75 C
26	36 B	64	27 C	94	78 C*
27	43 C	65	51 C	95	76 C
29	41 C	66	55 C	96	79 C*
32	27 B	67	53 C*	98 I.	62 C
35 I.	31 C	68	54 C	98 II.	63 C
35 II.	32 C	69	56 C	99	30 C
41	32 B	70	57 C		
42	31 B	71 I.	58 C		

* Cases of only approximate correspondence.

This and the preceding lists of experiments have been prepared by the author with the view of satisfying both the letter and the spirit of the latest Harvard requirements (1890–1891). In view, however, of the frequent and extensive changes which have taken place in these requirements, teachers will do well to consult members of the physical department before deciding what particular experiments they propose to have their pupils offer for admission to the University.

APPENDIX IX.

AVERAGES OF VARIABLE QUANTITIES.

The average value of a variable quantity is frequently required in physical measurement, as, for instance, in cases where the *average* atmospheric temperature or pressure affect the results. If a sufficient number of observations be taken there is no especial difficulty in computing their average; but the average of a variable quantity can be found in general only through formulæ established by the differential and integral calculus. It is doubtful whether experiments involving the use of such formulæ should be included in an elementary course; but if included, every effort should be made to explain the formulæ to the student.

The teacher may, in certain cases, find it advisable to anticipate some of the principles of the calculus rather than to defer an experiment until these principles would naturally be explained. This, however, is not generally necessary.

It will be seen from the following demonstrations that the averages of variable quantities may be obtained in a great many cases by simple arithmetic, algebraic, or geometric processes *without the aid of the calculus*, and when so obtained can be tabulated and employed in place of the integrals to which they correspond.

It is important to present new problems in their simplest possible form. The ideas involved in processes of averaging are not only more familiar, but also simpler than in integration; for the integral is a quantity which necessarily (unlike the average) *differs in kind* from the quantity operated upon. The use of averages will be found, accordingly, to have certain marked advantages over the use of integrals for the purposes of elementary demonstration.

(*a*) *Numerical Averages.* The average of a given number of terms is defined as the sum of the terms divided by that number. It may be assumed that students are already familiar with arithmetical processes by which averages are obtained. The same processes may be extended to cases in which it is desired to find the average of numerical functions, provided of course that all the values to be averaged are finite, and limited in number.

The average of all integral numbers between 0 and 10 inclusive is, for instance, 5; the average of the squares of these numbers is 35; the average of the cubes of these numbers is 275. A slightly different result would be obtained if intermediate values of these functions were also averaged. If, for instance, every integral number of tenths were considered, the average of the numbers would be 5 as before; but the average of the squares would be 33.5, and the average of the cubes 252.5.

If now we should consider every integral number of hundredths, the average would become 33.3 + and 250 + respectively. The same would be true if we considered thousandths or millionths of a unit. It would appear, accordingly, that the numbers which we have found represent with an increasing degree of accuracy the average value of the square and the cube of a quantity varying by small but equal steps between the values 0 and 10.

(*b*) *Limits of Error.* The truth of this statement is capable of demonstration. The average value of the square of a quantity between 0 and 1 cannot, for instance, be greater than 1 (the maximum value), nor less than 0 (the minimum value). In the same way the average value of the square of all numbers between 1 and 2 cannot be greater than 4 nor less than 1, &c. It follows that the mean square of a continuous variable between the values 0 and 10 cannot be greater than the average of the squares 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100, nor less than the average of the squares 0, 1, 4, 9, 16, 25, 36, 49, 64, and 81. That is, this mean square is necessarily greater than 28.5, and less than 38.5; hence equal to 33.5 *within* 5 *units.*

In the same way, by considering all possible numbers between 0 and 10 which can be expressed by an integral number of units and tenths, it can be proved that the mean square in question is equal to 33.335 *within* 5 *tenths of a unit;* and a still closer approximation is obtained by considering averages through intervals of one hundredth of a unit each.

The mean cube of a continuous variable between given limits can be found in the same way with any required degree of accuracy by purely arithmetical processes. The same methods are applicable to the case of any function whatsoever — always excluding the case of infinite or imaginary values. This fact may be made use of for the purpose of demonstrating to a class in elementary physics the value of certain mathematical constants which are usually determined only by the aid of higher mathematics. An example will be found in section *g* of Appendix X., relating to Probable Error, in which the "Coefficient of Probability" is determined roughly in this way.

(*c*) *Average of a variable x.* The use of purely arithmetical processes is confined to cases in which quantities are to

be averaged between given limits. It has been shown, for instance, that the average of all the numbers between 0 and 10 inclusive is 5. It will be found that the average of all numbers between 0 and 100 inclusive is 50, &c. The question naturally arises, is the average of all numbers between 0 and a given number *always* equal to one half of the given number?

Let us call the number n; then there are n terms to be averaged. The first is 0; the last is n; these two give an average of $(n + 0) \div 2 = \frac{1}{2} n$. The second (1) and next to the last $(n - 1)$ give similarly an average $((n - 1) + 1) \div 2 = \frac{1}{2} n$. The numbers can evidently be thus combined in pairs, each averaging $\frac{1}{2} n$, until, if n is even, all the numbers are paired off; or if n is odd, a single number $(\frac{1}{2} n)$ remains. Obviously the average of all these averages is in any case $\frac{1}{2} n$; hence this must be the average of all the numbers.

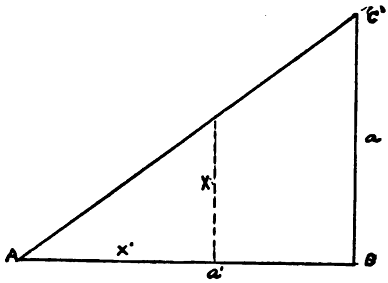
The same result would be obtained if we considered tenths or hundredths of a unit. We should have a greater number of pairs to be averaged, but as the value of each is $\frac{1}{2} n$, the average would be the same. We conclude, therefore, that the average value of a variable x between the value 0 and n is always equal to $\frac{1}{2} n$.

(*d*) *Notation of Averages.* The result obtained in the last section may be expressed as follows: —

$$\overset{o \,\rule[0.5ex]{2em}{0.4pt}\, n}{x} = \tfrac{1}{2} n. \qquad (1)$$

The line placed above the letter x denotes that an average is to be taken, and the limits of this average are indicated by the two values placed one at each end of the line. In such expressions variable quantities are customarily denoted by letters near the end of the alphabet (especially x, y, and z), while other letters, like numerals, denote constant quantities.

(*e*) *Geometrical Proof.* The area of a rectangle is found by multiplying together the base and altitude of the rectangle. The areas of other figures may similarly be found by multiplying together the base and *average* altitude of the figures. Thus the area (A) of an isosceles right-angle triangle ABC is equal to the product of the base ($AB = a' = a$) and its average altitude $x = x'$. That is:



$$A = \overset{o\text{———}a}{x} \times a'$$

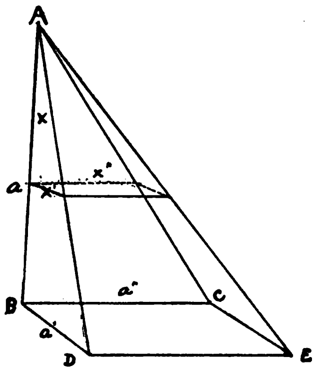
Now by geometry —

$$A = \tfrac{1}{2}\, a \times a';$$

hence, as before —

$$\overset{o\text{———}a}{x} = \tfrac{1}{2}\, a.$$

(*f*) *Average of* x^2. The volume of a rectangular block is found by multiplying together the cross-section and altitude of the block. In the same way the volume of other figures may be found by multiplying together the *average* cross-section and the altitude of these figures. Let the altitude of a pyramid $ABCDE$ be a, and its base $a' \times a'' = a^2$; then the cross-section of the pyramid at a distance x from the apex (A) is $x' \times x'' = x^2$. The volume V is therefore —



$$V = \overset{o \,\text{——}\, a}{x^2} \times a.$$

Now by geometry —

$$V = \tfrac{1}{3}\, n^2 \times a; \text{ hence}$$

$$\overset{o \,\text{——}\, a}{x^2} = \tfrac{1}{3}\, a^2. \qquad (2)$$

We have already seen that the average of the squares of numbers from 0 to 10 is approximately equal to 33⅓, and that when the squares are taken closer and closer together, there is a still closer approach to this value. The formula (2) shows that when all possible intermediate values are considered, the average is exactly 33⅓. It also shows that in general the average value of the square of a quantity up to a given value is equal to one third of the square of this value.

(*g*) *Mechanical Proof.* To find the volume of a small prism, we multiply its length (l) by its (uniform) cross-section (q). To find the mass (m) of the prism, we multiply the result by the (uniform) density of the prism (d). To find the moment (M) of the prism about a distant point (o) in line with the axis of the prism, we multiply the result by the (nearly constant) arm (a) in question. That is —

$$M = l \times \bar{q} \times \bar{d} \times \bar{a} \text{ (nearly).}$$

Now if c is the distance from the centre of gravity to the point o, the moment of the prism about o is by definition —

$$M = m \times c;$$

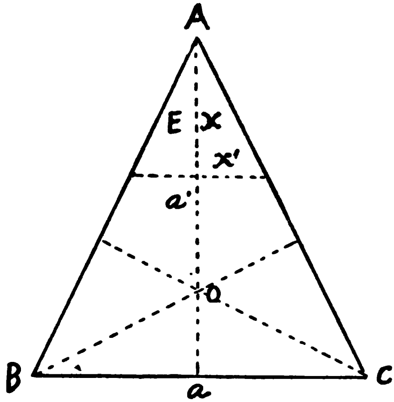
hence we have —

$$m \times c = l \times \bar{q} \times \bar{d} \times \bar{a}; \text{ or}$$

$$c = \bar{q} \times \bar{d} \times \bar{a} \times l \div m.$$

In a similar manner, the centre of gravity of any figure with respect to a given axis, is located by multiplying the quotient of the length of the figure by the mass into the *average* product of the cross-section, density, and arm corresponding to regularly increasing distances along the axis.

The distance from the apex A to the centre of gravity (O) of an isosceles triangle ABC, with base a and altitude a, and unit thickness and density throughout is found, accordingly, by averaging the products of the distances x from the apex by the cross-section, also equal to x, multiplying by the (vertical) length a, and dividing by the mass, $\frac{1}{2}a^2$. That is —



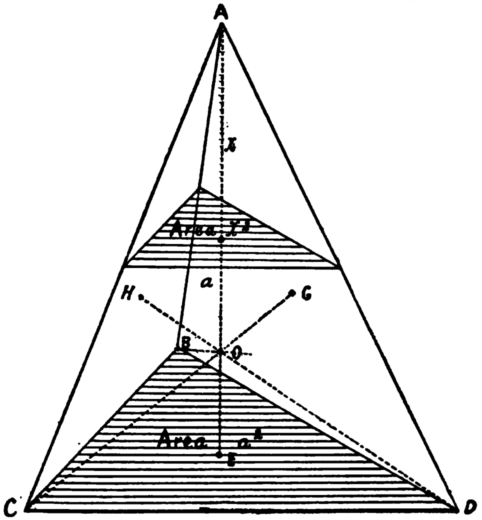
$$AO = \overset{o \text{———} a}{x^2} \div \left(\tfrac{1}{2}a^2 \div a\right) = \overset{o \text{———} a}{x^2} \div \tfrac{1}{2}a.$$

Now the centre of gravity of a triangle must lie at the common intersection of three lines, AO, BO, and CO, which bisect the sides BC, AC, and AB; for if the triangle be cut up into a series of bars parallel to either of the sides, each bar will balance about an axis which bisects it; hence the triangle as a whole must balance about such an axis, and this axis must contain the centre of gravity. Now the three lines in question can be shown by geometry to intersect at a point O, such that $AO = \frac{2}{3}a$. Hence we have, substituting, —

$$\overset{o\,\text{——}\,a}{x^2} \div \tfrac{1}{2}\,a = \tfrac{2}{3}\,a,$$

or as before $$\overset{o\,\text{——}\,a}{x^2} = \tfrac{1}{3}\,a^3.$$

(*h*) *Average of* x^3. In the same way the centre of gravity (O) of a pyramid $ABCD$, with altitude a, base a^2, and



density 1, is found by multiplying together the distances x and the cross-section x^2, and dividing the result by the ratio of the mass ($\frac{1}{3}\,a^3$) to the altitude (a). That is,

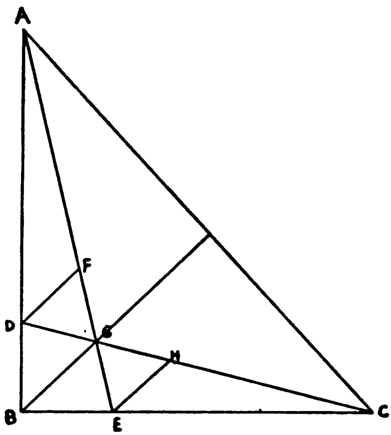
$$AO = \overset{o\,\text{——}\,a}{x^3} \div (\tfrac{1}{3}\,a^3 \div a) = \overset{o\,\text{——}\,a}{x^3} \div \tfrac{1}{3}\,a^2.$$

Now a pyramid must balance about any one of the four axes, AO, BO, CO, or DO, passing through the centre of gravity of the four sides, and hence through that of every section parallel to their sides; and since by geometry $AO = \frac{3}{4}\,a$, we have substituting—

$$\underset{x^3}{o \text{———} a} \div \tfrac{1}{3} a^2 = \tfrac{3}{4} a; \text{ or}$$

$$\underset{x^3}{o \text{———} a} = \tfrac{1}{4} a^3. \qquad (3)$$

(*i*) *Average of* x^n. Let ABC be an isosceles right triangle, with density 0 along the line AC and density x^{n-1} at a distance x from AC measured *either* horizontally or vertically; then we have seen that the centre of gravity of a horizontal or vertical section of the triangle is (if $n = 1$, 2, or 3) at a point such that the distance between it and AB or BC is to the distance between it and AC as $1 : n$. The centres of gravity of all sections lie, therefore, on the lines AE or CD, dividing the sides BC and AB in the same proportion, so that —



$$BE : EC :: BD : DA :: 1 : n.$$

The centre of gravity of the triangle is therefore at the intersection G of AE and CD. Drawing BG and, parallel to it, EH and DF, we have by similar triangles —

$$GH : HC :: GF : FA :: 1 : n.$$

Now by construction the triangles HGE and DGF are equal; hence $GE = GF = FA \div n$, and $DG = GH = HC \div n$.

It follows that the distance of the centre of gravity from the base of the triangle is to the distance of the apex from the base as $GE : AE$, or as $GE : AF + FG + GE$, or as

$$\frac{1}{n} : 1 + \frac{1}{n} + \frac{1}{n}, \text{ or as } 1 : n + 2.$$

Denoting the altitude of the triangle by a, we have, therefore, fo- the vertical distance d of the centre of gravity from the apex —

$$d = \frac{n+1}{n+2}\, a.$$

Now applying the ordinary methods we find the average density of a horizontal cross-section to be —

$$\overset{o\,\text{———}\,x}{x^{n-1}} = \frac{x^{n-1}}{n}$$

which multiplied by the cross-section (x) gives $\frac{x^n}{n}$ for the mass of the cross-section. The total mass is therefore (if $n = 1$, 2, or 3) —

$$a \times \overset{o\,\text{———}\,a}{\frac{x^n}{n}} = \frac{a^{n+1}}{n(n+1)}.$$

The ratio of the mass to the altitude is —

$$\frac{a^{n+1}}{n(n+1)} \div a = \frac{a^n}{n(n+1)}.$$

The moment of a given section about the apex is —

$$x \times \frac{x^n}{n} = \frac{x^{n+1}}{n}.$$

The distance of the centre of gravity is accordingly —

$$d = \frac{\overset{o\,\text{——}\,a}{x^{n+1}}}{n} \div \frac{a^n}{n\,(n+1)} = \frac{n+1}{n+2}\,a.$$

It follows that—

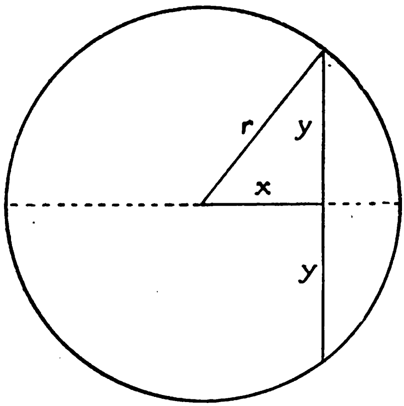
$$\overset{o\,\text{——}\,a}{x^{n+1}} = n \times \frac{a^n}{n\,(n+1)} \times \frac{n+1}{n+2}\,a = \frac{a^{n\ \ 1}}{n+2}.$$

It has been proved that if $n = 1, 2$, or 3,

$$\overset{o\,\text{——}\,a}{x^{n}} = \frac{a^n}{n+1};$$

it follows that the same expression holds good for x^{n+1}; hence it holds for x^4, hence for x^5, &c. In other words, we have in general (for positive integral powers of n),

$$\overset{o\,\text{——}\,a}{x^{n}} = \frac{a^n}{n+1}. \qquad (4)$$



(*j*) *Average of Trigonometric Functions.* Passing now to the case of a circle of radius, r, and area, A, which we con-

sider as the product of its diameter ($2r$) and *average* cross-section ($2\,y = 2\,\sqrt{r^2 - x^2}$), we find—

$$A = \pi\,r^2 = 2\,r \times {}^{-r}\overline{\;2\,\sqrt{(r^2 - x^2)}\;}{}^{+r}$$

whence, dividing through by $4\,r$,

$$ {}^{-r}\overline{\;(r^2 - x^2)^{\frac{1}{2}}\;}{}^{+r} = \frac{\pi\,r}{4} \qquad (5)$$

Again, from the properties of the circle we have $\sin^2 x + \cos^2 x = 1$; hence

$$\overline{\sin^2 x + \cos^2 x} = 1.$$

Now $\sin 0° = \cos 90°$; $\sin 1° = \cos 89°$, &c. Hence,

$$ {}^{0°}\overline{\;\sin^2 x\;}{}^{90°} = {}^{0°}\overline{\;\cos^2 x\;}{}^{90},$$

and we have—

$$ {}^{0°}\overline{\;\sin^2 x\;}{}^{90°} = {}^{0°}\overline{\;\cos^2 x\;}{}^{90°}$$

$$ = {}^{0°}\overline{\;\sin^2 x + \cos^2 x\;}{}^{90°} = \tfrac{1}{2} \qquad (6)$$

The results already obtained are sufficient to illustrate certain methods by which functions may be averaged without the aid of the calculus when, through geometrical construction or otherwise, the averages of similar functions are known.

APPENDIX X.

PROBABILITY OF ERRORS.

(*a*) *Definitions.* The observed value (o) of a quantity differs, as has been pointed out in § 156, from the true value (q) by an amount (e) which is called the error of observation. Let the errors in a series of n observations be distinguished by subscript numerals, then the average error, $\bar{e}$, is defined by the equation

$$\bar{e} = [\Sigma](e_1 + e_2 + e_3 + \ldots + e_n) \div n, \quad (1)$$

where the sign $[\Sigma]$ indicates that the numerical values of the errors are to be added together without regard to algebraic signs.

The mean square of the errors ($\overline{e^2}$) is defined by the equation

$$\overline{e^2} = \Sigma\,(e_1^2 + e_2^2 + e_3^2 + \ldots + e_n^2) \div n. \quad (2)$$

The "error of the mean square" (ϵ) is defined simply as the square root of the mean square of the errors; that is

$$\epsilon = \sqrt{\overline{e^2}} = \sqrt{\Sigma(e_1^2 + e_2^2 + e_3^2 + \ldots + e_n^2) \div n}. \quad (3)$$

In a long series of observations in which the errors are due to a great number of causes combining together in every possible way, the "probable error" is defined as one which is neither greater nor less than the majority. That is, if the errors are arranged in the order of their magnitude (without regard to signs), as follows, —

$$e_0, e_1, e_2, e_3 \ldots e_p, e_{p+1}, e_{p+2}, e_{p+3}, \ldots e_n,$$

the "probable error," e_p, is defined by the equation —

$$p = \tfrac{1}{2}\, n. \qquad (4)$$

The "coeficient of probability," π, is defined as the ratio of the prolable to the mean error; that is,

$$e_p = \pi\epsilon. \qquad (5)$$

This definition is customarily extended to the case of short series of observations. In such cases the equation serves, however, to define the probable error, not the coefficient of probability. The value of this coefficient will be determined roughly in the course of the following elementary investigation (see (*g*)).

(*b*) *Distribution of errors.* The simplest case in the theory of errors is one in which practically a single source of error exists. Let us take as an example the indication of a spring balance which (from rust or other causes) is affected by friction much more than by any other cause. If the pointer of such an instrument be pulled down to a given reading, the reading is generally too small. If the pointer is first pulled down too far, then allowed to recover, the indication is generally too great. We will suppose for simplicity that the error is 1 unit in each case, — a kilogram for instance; then the average error, the mean square of the errors, and the error of the mean square, are all by definition equal to 1.

Now let it be required to find the weight of a body which rests, as in ¶ 159 (1) upon two such spring balances. Let us suppose that each balance is pulled down to a given reading in one half of the observations, but allowed to recover to the reading in the other half of the observations. Let us suppose, moreover, that the two balances are treated first in the same way, then in opposite ways. It follows that in one half of the observations the errors due to friction in the bal-

ances will offset each other; but that in the other half, they will combine together so as to give a resultant $\pm$ 2.

In point of fact, when a weight is thrown upon a spring balance, it usually makes several oscillations before it comes to rest. There is accordingly a (nearly) equal chance that the pointer may be arrested by friction either above or below its mean resting-point. We may assume, therefore, that in (about) half of the observations the readings will be increased, in the other half diminished by friction. Let us first consider those observations in which the readings of one spring balance are too great. In the absence of any necessary reason why the readings of the second spring balance should be affected by those of the first, we may assume that these readings are also too great in about half the cases under consideration, — that is, one fourth of the total number of cases, — and too small in the remaining fourth. In the same way, by considering that half of the observations in which the readings of the first spring balance are too small, we find a third fourth in which the readings of the second spring balance are also too small, and a fourth fourth in which they are too great. We find as before that the errors offset each other in one half (or two fourths) of the observations, and that they combine together in the other half. That is, the distribution of errors due to chance is (nearly) the same as that in which care is taken to bring about with equal frequency every possible combination.

The four ways in which two sources of error, each equal to $\pm$ 1, may combine together, may be written as follows:

$$A = +1 + 1 = +2; \qquad C = -1 + 1 = 0;$$
$$B = +1 - 1 = 0; \qquad D = -1 - 1 = -2.$$

Each of these combinations differs from the next simply in the fact that the sign of *one* error is reversed. It is always assumed in the treatment of accidental errors that positive

and negative errors are equally probable.[1] It follows that each of the combinations named above is just as probable as the next; hence all are equally probable.

Now let us consider a third source of error, — a body, for instance, suspended in parts from three spring balances. Each of the four combinations named above gives two, in one of which the resultant is increased, in the other diminished by the new source of error. There are accordingly 8 combinations in all, namely,

$$\begin{array}{ll}
a = A + 1 = +3; & e = C + 1 = +1; \\
b + A - 1 = +1; & f = C - 1 = -1; \\
c = B + 1 = +1; & g = D + 1 = -1; \\
d + B - 1 = -1; & h = D - 1 = -3;
\end{array}$$

In one case the resultant is $+3$, in 3 cases $+1$, in 3 cases -1, in 1 case -3.

We come next to four sources of error. Each of the previous 8 combinations yields 2, so that there are 16 in all, namely,

$$\begin{array}{llll}
a + 1 = 4 & c + 1 = 2 & e + 1 = 2 & g + 1 = 0 \\
a - 1 = 2 & c - 1 = 0 & e - 1 = 0 & g - 1 = -2 \\
b + 1 = 2 & d + 1 = 0 & f + 1 = 0 & h + 1 = -2 \\
b - 1 = 0 & d - 1 = -2 & f - 1 = -2 & h - 1 = -4.
\end{array}$$

In 1 case we have a resultant $+4$, in 4 cases $+2$, in 6 cases 0, in 4 cases -2, in 1 case -4.

(*c*) *Table of Combinations.* In the same way, by purely arithmetical processes, the distribution of errors due to any number of sources may be obtained. The results already calculated for the case of 4 sources of error are compared in

[1] If this were not the case we should have a constant error as the result. It is supposed that all *constant* errors are eliminated.

the table below with similar results corresponding respectively to 16 and to 100 sources of error.

Magnitude of the Error.	No. of combinations giving a resultant of this magnitude when the No. of sources = 4.	No. of sources = 16.	No of sources = 100.	Magnitude of the Error.
0	6	12870	1009×10^{26}	0
+2	4	11440	989 "	—2
4	1	8008	932 "	—4
6	0	4368	844 "	—6
8	0	1820	734 "	—8
10	0	560	614 "	—10
12	0	120	494 "	—12
14	0	16	381 "	—14
16	0	1	282 "	—16
18	0	0	201 "	—18
20	0	0	137 "	—20
22	0	0	90 "	—22
24	0	0	57 "	—24
26	0	0	35 "	—26
28	0	0	20 "	—28
30	0	0	11 "	—30
32	0	0	6 "	—32
34	0	0	3 "	—34
36	0	0	1+ "	—36
38	0	0	1— "	—38
+40	0	0	0+ "	—40
Total	16	65,536	$12{,}677 \times 10^{26}$	Total

(*d*) *Probability Curve.* The results contained in the last table are represented graphically in the figure. In constructing this figure, the vertical distances were made proportional to the number of combinations in a given column, the first number in the column being taken equal to 1. The horizontal distances were made proportional to the magnitude of the resulting errors represented by a given curve; but in plotting the first curve, the results were divided by 2, in the second by 4, in the third by 10. The similarity of the curves when thus reduced to a common scale becomes apparent.

The three curves in the figure represent the relative probability of errors of a given magnitude resulting (1) from combinations of 4 sources, each equal to $\pm \frac{1}{2}$; (2) from combinations of 16 sources, each equal to $\pm \frac{1}{4}$; and (3) from combinations of 100 sources, each equal to $\pm \frac{1}{10}$, — because, in plotting the curves, we divided the resultants by 2, 4, and 10, respectively. The (approximate) agreement of the curves serves, therefore, to illustrate the truth of a general law, that the distribution of errors due to n sources, each equal to $\pm 1 \div \sqrt{n}$, is in all cases (approximately) the same.

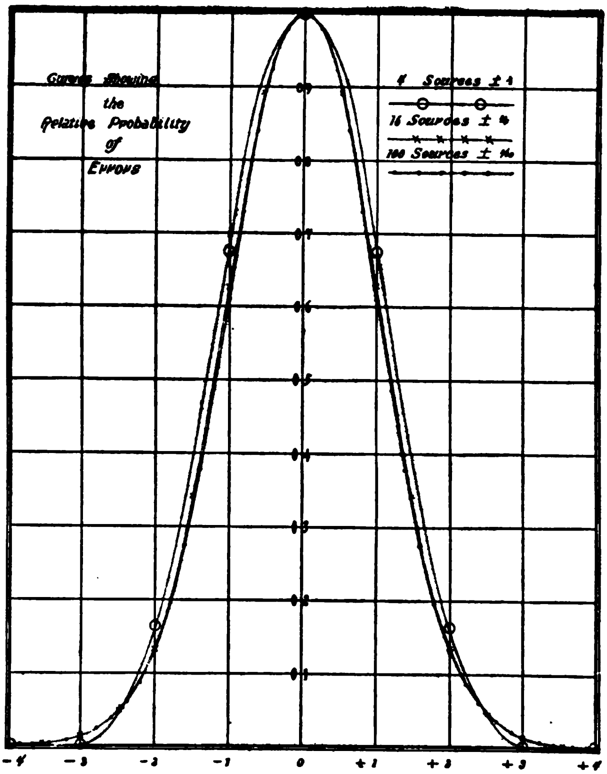
It follows from this law that the average error and the error of mean square must also, under the conditions named, be (approximately) the same. It also follows that the resultants due to n *unit* sources of error are, other things being equal, proportional (approximately) to the square root of n. Hence the average error and the error of the mean square are also (approximately) proportional to the square root of the number of sources from which they arise. This law, which holds only approximately for average errors, can be proved exactly in the case of the error of the mean square.

(*e*) *Calculation of Mean Squares.* The results which have been worked out in the distribution of errors enable us to calculate in certain cases the mean square of these errors. When a single unit source of error exists, the resultant is always ± 1, hence the mean square is $+1$, or $\epsilon_1^2 = 1$. With 2 unit sources, we have four combinations, in two of which the resultant is 0, in the other two ± 2. The mean square is accordingly

$$\epsilon_2^2 = (2 \times 0 + 2 \times 4) \div 4 = 2.$$

In the same way, in the case of three unit sources, we find

$$\epsilon_3^2 = (2 \times 9 + 6 \times 1) \div 8 = 3.$$


Curves Showing
the
Relative Probability
of
Errors
4 Sources
16 Sources
100 Sources
-4
-3
-2
-1
0
+1
+2
+3
+4

and in the case of four unit sources —

$$\epsilon_4^2 = (2 \times 16 + 8 \times 4 + 6 \times 0) \div 16 = 4.$$

We notice that the mean square of the resultant error is in each case equal to the number of unit sources of which it is composed.

It is easy to show that this law is perfectly general. Suppose we have found the distribution of errors due to n unit sources to be such as to give

x resultants of the magnitude e,
y resultants of the magnitude f, *&c.*,

and that a new unit source of error is now added, so as to increase half of the previous resultants by 1, and to diminish the other half by 1. We shall then have

$$\begin{cases} \tfrac{1}{2}\, x \text{ resultants of the magnitude } e + 1 \\ \tfrac{1}{2}\, x \text{ resultants of the magnitude } e - 1 \end{cases}$$
$$\begin{cases} \tfrac{1}{2}\, y \text{ resultants of the magnitude } f + 1 \\ \tfrac{1}{2}\, y \text{ resultants of the magnitude } f - 1, \text{ \&c.} \end{cases}$$

The squares of these errors will be, in

$$\begin{cases} \tfrac{1}{2}\, x \text{ cases, } e^2 + 2\,e + 1 \\ \tfrac{1}{2}\, x \text{ cases, } e^2 - 2\,e + 1, \text{ \&c.,} \end{cases}$$

the average in the x cases being $e^2 + 1$.

In the same way the average in y cases is $f^2 + 1$, &c. The mean square of all the errors is therefore

$$\begin{aligned} \epsilon^2_{n+1} &= [x\ (e^2 + 1) + y\ (f^2 + 1) + \text{\&c.}] \div [x + y + \text{\&c.}] \\ &= [x\,e^2 + y\,f^2 + \text{\&c.} + x + y + \text{\&c.}] \div [x + y + \text{\&c.}] \\ &= [x\,e^2 + y\,f^2 + \text{\&c.}] \div [x + y + \text{\&c.}] + 1. \end{aligned}$$

Now if it has been shown that in the case of n sources of error, the mean square of the resultant is equal to n, or if

$$\epsilon^2_n = [x e^2 + y f^2 + \&c.] \div [x + y + \&c.] = n,$$

we have, substituting,

$$\epsilon_{n+1}^2 = \epsilon_n^2 + 1 = n + 1.$$

The law has been shown to hold in the case of four sources of error, hence it holds for 5, hence for 6, &c.; that is, it holds for any number of sources of error.

We have therefore, the following law: —

The mean square of the errors resulting from every possible combination of a given number of unit sources is equal to the number in question.

(f) *Law of Mean Squares.* If, instead of combining an error $\pm$ 1 with x errors of the magnitude e, we should combine an error of the magnitude $\pm$ e' with them, the mean square of the resultant would be $e^2 + (e')^2$.

If therefore a new source of error arises in which the resultants are in x' cases, $\pm$ e', in y' cases, $\pm$ f', &c., the mean square of the resultant will be

$$[x' (e^2 + (e')^2) + y' (e^2 + (f')^2) + \&c.] \div [x' + y' + \&c.]$$
$$= [x' e^2 + y' e^2 + \&c. + x' (e')^2 + y' (f')^2 + \&c.] \div [x' + y' + \&c.]$$
$$= e^2 + [x' (e')^2 + y' (f')^2 + \&c.] \div [x' + y' + \&c.]$$
$$= e^2 + \epsilon^2,$$

where ϵ^2 represents the mean square of the resultant errors due to the new source. In the same way, the mean square of the resultant due to combining the new source of error with the resultants f, is to make the mean square of these resultants, $f^2 + \epsilon^2$; that is, the squares of all the previous resultants are increased on the average by the amount ϵ^2; hence also the mean square of these resultants.

The law of mean squares may now be stated in general as follows: —

The mean square of the errors resulting from combinations of accidental causes is equal to the sum of the mean squares due to each separate cause.

(g) *Coefficient of Probability.* The coefficient of probability, or ratio of the probable error to the error of the mean square depends upon the manner in which errors due to a great variety of sources are distributed in a long series of observations. A calculation of this coefficient may be based upon any of the curves plotted in the figure (see d). We will choose the curve due to 100 sources of error, not only because this curve is better defined than the others, but also because it has been found not to differ perceptibly from curves obtained with 1,000 or 1,000,000 sources of error. There is in fact a wide limit within which the results may be applicable. At the same time it must not be forgotten that the other curves yield results which in a great many cases would be more conformable to the facts.[1]

We find from the table (see c) that the number of combinations due to 100 unit sources which give resultants less than $\pm$ 6 is,

$$(932 + 989 + 1009 + 989 + 932) \times 10^{23} = 4851 \times 10^{26}.$$

In the same way we find the following numbers: —

Less than	$\pm$ 6,	4851×10^{26}	Less than	$\pm$ 8,	6539×10^{26}
Equal to	$\pm$ 6,	1688 "	Equal to	$\pm$ 8,	1468 "
Greater than	$\pm$ 6,	6138 "	Greater than	$\pm$ 8,	4670 "
Total		12677×10^{26}	Total		12677×10^{26}

[1] Most observations are affected by a few large sources of error, in comparison with which the smaller sources may be neglected. The distribution of errors may also be altered by the nature of the sources from which they arise.

Evidently the probable error is greater than $\pm$ 6, because there are more combinations giving a greater than a smaller resultant. The probable error is also evidently less than $\pm$ 8. It would be sufficiently accurate for most purposes to call the probable error in such a case 7 units. A more precise value may be found by interpolation.

In practice resultant errors are not confined to certain definite values such as $\pm$ 6 or $\pm$ 8, with gaps between them, but we find them, as indicated by the continuous curves of the figure, more or less uniformly distributed. We may assume therefore, as a first approximation, that the 1468×10^{26} combinations which in a hundred sources of error each equal to $\pm$ 1 would give resultants equal to $\pm$ 8, are in practice distributed evenly between $\pm$ 9 and $\pm$ 7; while the 1688×10^{26} resultants of $\pm$ 6 reach from $\pm$ 7 to $\pm$ 5. There would in this case be about 844×10^{26} combinations between $\pm$ 6 and $\pm$ 7, and $(844 + 4851) \times 10^{26}$, or 5695×10^{26} combinations below $\pm$ 6. The probable error (e_p or p) corresponds to half the total number of combinations, that is, to $\frac{1}{2}\, 12677 \times 10^{26} = 6338 \times 10^{26}$, nearly. Applying the ordinary rules for interpolation,[1] we find

$$p = 6 + (6338 - 5695) \div 844 = 6.7 +.$$

Now the error of the mean square due to 100 unit sources of error is, according to section (*e*), equal to $\sqrt{100}$ or 10 units; hence the coefficient of probability is 0.67 +.

(*h*) *Probable Errors of Sums and Differences.* When several quantities are added together, the sum is affected by

[1] A more exact value of this coefficient may be obtained by methods of interpolation involving 2d and 3d differences, or by considering a greater number of sources of error. Such a value of the coefficient is 0.67449. A similar investigation shows that the ratio of the probable to the average error is 0.8453.

the errors in each quantity. Since the probable error is always proportional to the error of mean square, we find, from the law of mean squares (see (f)), that the probable error (p) of the sum is equal to the square root of the sum of the squares of the probable errors (p_1 p_2, &c.) of all its components; that is,

$$p = \sqrt{p_1^2 + p_2^2 + \&c.}$$

The case is somewhat simpler when all the quantities added together are affected by the same sources of error. The number of sources of error in the result is then proportional to the number of the quantities, hence also the mean square of the errors. It follows that the probable error is proportional in such cases to the square root of the number of terms added together.

The same principles apply to the calculation of resultant errors in differences as in sums; for negative errors are supposed to be just as frequent as positive errors, hence it can make no difference in a long series of results, as far as the *errors* are concerned, whether one quantity is to be added or subtracted from another.

The probable error of the sum or difference of two quantities affected by the same sources of error is therefore greater than the probable error of the quantities themselves in the ratio of $\sqrt{2}$ to 1. Evidently the error bears a greater proportion to the difference of the two quantities than to their sum. Methods of difference are, therefore, relatively inaccurate (see § 38).

(i) *Multiplication and Division of Errors.* When an observed quantity is multiplied or divided by a constant, the error of observation is also evidently multiplied or divided by the same constant; hence the probable error is increased or diminished in the same proportion.

(*j*) *Probable Errors of Averages.* If the probable error of a single observed quantity is p, that of the sum of n similar quantities is, as we have seen (see (h)), $p\sqrt{n}$. In finding the average of n quantities, we divide their sum by n; hence, according to the last section, the probable error is also divided by n. It follows that the probable error of the average of n similar observations is $p\sqrt{n} \div n = p \div \sqrt{n}$. That is, *the probable error of the average of several observations affected by like sources of error varies inversely as the square root of the number of observations.*

(*k*) *Estimation of probable error from the differences between separate observations and their mean.*

Let $q + e_1$, $q + e_2$, $q + e_3$ . . . $q + e_n$ denote a series of n observations of a quantity q. The mean (m) of these observations is then

$$m = q + (e_1 + e_2 + e_3 + \ldots + e_n) \div n.$$

The difference (d_1) between the first observation and the mean is

$$d_1 = q + e_1 - m = e_1 - (e_1 + e_2 + e_3 + \ldots + e_n)\, n$$
$$= \frac{n-1}{n} e_1 + \frac{e_2}{n} + \frac{e_3}{n} + \ldots \frac{e_n}{n}.$$

That is, the difference between a given observation and the mean is found by multiplying one of the errors by $(n - 1) \div n$, and the other $(n - 1)$ errors by $1 \div n$. If ϵ^2 is the mean square of the errors, the mean square of one of the terms must be $\epsilon^2 (n-1)^2 \div n^2$; while the mean square of the other $(n - 1)$ terms will be $\epsilon^2 \div n^2$; so that the sum of these mean squares, which is equal to the mean square of the differences

$$\overline{d^2} = \frac{\epsilon^2 (n-1)^2}{n^2} + (n-1)\frac{\epsilon^2}{n^2} = \frac{n-1}{n}\epsilon^2. \quad (1)$$

Conversely we have

$$\epsilon^2 = \frac{n}{n-1}\,\overline{d^2}. \tag{2}$$

From this and preceding formulæ, we find the following rule for the calculation of probable error: Add all the observations together, divide by their sum to find the mean, subtract the mean from each observation to find the "differences;" square all these differences, add the squares together, divide by the number of observations less one, to find the mean square of the error of observation; extract the square root of this result to find the error of the mean square, multiply the error of the mean square by 0.67449 (0.7, *nearly*) *to find the probable error of observation; divide by the square root of the number of observations to find the probable error of the mean of the observations.*

(*l*) *Example of the calculation of probable error* (see § 50).

A. No. of observations.	B. Boiling point observed.	C. Differences from the average.	D. Squares of the differences.
1	78°.79	+0°.29	0.0841
2	78.33	−0.17	289
3	78.02	−0.48	2304
4	78.93	+0.43	1849
5	78.46	−0.04	16
6	78.67	+0.17	289
7	78.00	−0.50	2500
8	78.81	+0.31	961
9	78.43	−0.07	49
10	78.56	+0.06	36
Sum	10) 785.00	10) [2.52]	9) 0.9134
Average	78.500	[0.252]	0.1015

Error of the mean square, $\sqrt{0.1015} =$ 0.318 +
Probable error of observation, (0.67 +) × (0.318 +) = 0.21 +
Probable error of the mean, 0.21 ÷ $\sqrt{10}$ = 0.21 ÷ 3.16 = 0.07
Final result for the boiling-point of alcohol, 78°.50 ± 0°.07

(*m*) *Probable errors of products and quotients.* The probable errors of products and quotients are easily calculated by considering the proportion which the component and resultant errors bear to the quantities which they affect. We have, for instance, in the notation previously employed (§ 156): —

$$o_1 \, o_2 = q_1 \times q_2 \times \left(1 + \frac{e_1}{q_1}\right)\left(1 + \frac{e_2}{q_2}\right) =$$

$$q_1 \times q_2 \times \left(1 + \frac{e_1}{q_1} + \frac{e_2}{q_2}\right) \text{ nearly,}$$

$$\frac{o_1}{o_2} = \frac{q_1}{q_2}\left(1 + \frac{e_1}{q_1}\right) \div \left(1 + \frac{e_2}{q_2}\right) = \frac{q_1}{q_2}\left(1 + \frac{e_1}{q_1} - \frac{e_2}{q_2}\right) \text{ nearly,}$$

neglecting in both cases terms which involve powers or products of the small ratios $(e : q)$; that is, neglecting terms of the second or higher degrees of smallness. It is evident from these formulæ that the proportional errors, $e_1 : q_1$, &c., are compounded in products and quotients just as ordinary errors are in sums and differences. The probable error (p) of a quantity q may, therefore, be calculated from the probable errors, p_1, p_2, p_3, &c., of its factors, q_1, q_2, q_3, &c., by the formula

$$\frac{p}{q} = \sqrt{\left(\frac{p_1}{q_1}\right)^2 + \left(\frac{p_2}{q_2}\right)^2 + \left(\frac{p_3}{q_3}\right)^2 + \text{\&c.}}$$

The same formula is to be employed whether the factors occur in the numerator or in the denominator of the fraction by which the quantity q is determined.

(*n*) *Probable Errors of Powers and Roots.* When an observed quantity, o, is raised to the power n, the result may be expressed: —

$$o^n = \left(q\left(1 + \frac{e}{q}\right)\right)^n = q^n\left(1 + \frac{e}{q}\right)^n =$$

$$q^n\left(1+\frac{ne}{q}+\frac{n\,(n-1)}{2}\,\frac{e_2}{q_2}+\&c.\right)=q_n\left(1+n\,\frac{e}{q}\right)\text{ nearly.}$$

The effect of raising an observed quantity to the power n is therefore to increase the proportional error, $e : q$, in the proportion $1 : n$. Since all such errors are increased in this proportion, the proportion which the probable error bears to the quantity which it affects must be increased in the same proportion.

The effect of extracting the n^{th} root of a quantity is the same as that of raising it to the power $1 \div n$; that is, the ratio of all errors, and hence that of the probable error to the quantities affected, is diminished in the ratio of $n : 1$.

(*o*) *General Method for finding the Probable Error of a Result.* The data from which results are calculated may generally be expressed in the form $q_1 = o_1 \pm e_1, q_2 = o_2 \pm e_2$, &c. The result r is then calculated from the values o_1, o_2, &c. Then the value $o_1 \pm e_1$ is substituted, and a new value of the result r_1 is calculated. Next the original value o_1 is employed, but $o_2 \pm e_2$ is substituted for o_2; and the corresponding value of the result r_2 is found, &c. The differences $d_1 = r_1 - r$; $d_2 = r_2 - r$, &c., represent the magnitude of the errors in the result due to the probable error in the several data. We have accordingly, from the law of mean squares, for the probable error (p) of the result

$$p = \sqrt{d_1^2 + d_2^2 + \&c.} \qquad (1)$$

(*p*) *Method for Determining the best possible Distribution of Time.* It is generally easy to see, from expressions for the probable error of a result, which of the data have the greatest influence upon the result. The number of observations upon which such data depend, should evidently be in-

creased, other things being equal, in preference to observations for the less important data. Let us suppose, however, that such observations are exceedingly difficult, or that the number already made is so great that little comparative advantage can be gained by spending upon them an additional (limited) amount of time. The question then arises, would it not be better to spend the *same* amount of time upon some of the less important data?

The question is one to which it is easy in most cases to give at least an approximate answer. First decide how much time can be spent; estimate from the results of experience how many observations of each kind can be made in this length of time. Calculate the diminution of the probable error in the case of each of the data due to the additional number of observations, and find as in the last section, the corresponding reduction in the probable error of the result. That distribution of time is of course the best which gives the greatest reduction in this probable error. Certain practical rules concerning the distribution of time will be found in § 49.

(*q*) *Method of Least Squares.* We have seen that the mean square of the errors of observation is an indication either of the number or of the magnitude of the sources of error. We make use of this fact in estimating the relative accuracy or inaccuracy of different methods of observation. That method is, other things being equal, the best which makes the sum of the squares of the errors the least.

In calculating errors we have to assume more or less knowledge of the true value of the quantity observed. Any error in such an assumption introduces a new (apparent) source of error into the observations. It therefore tends, on the whole, to increase (apparently) the mean squares of the errors. Of two assumptions, we choose therefore, other things

being equal, that which makes the sum of the squares of the errors of observation appear to be the least.

Let us take, for example, the case of a brass rod, the length of which was found to be 1000.0 *mm.* at 0°, 1001.7 *mm.* at 100°, and 1004.0 at 200°. The most probable value of the coefficient of expansion in such a case evidently lies between the maximum value observed (0.000023 from 100° to 200°) and the minimum value (0.000017 from 0° to 100°). The most probable value of the length of the rod at 0° lies moreover between 999.4 *mm.* (which would correspond to a length 1001.7 *mm.* at 100° and a coefficient of expansion 0.000023) and 1000.6 *mm.*; which would correspond to a length 1004.0 at 200° and a coefficient of expansion 0.000017. Let us assume that the length at 0° and the coefficient of expansion are half-way between these limits, that is 1000.0 *mm.* and 0.000020 respectively. The length at 0°, 100°, and 200°, should then be 1000.0, 1002.0, and 1004.0, respectively. This would make the errors of observation (expressed in tenths) 0, —3, and 0, respectively; hence the sum of the squares would be 9. The sum of the squares in this and other cases is shown in the table below:

	.000017	.000018	.000019	.000020	.000021	.000022	.000023
999.4	216	161	116	81	56	41	36
999.5	171	122	83	54	35	26	27
999.6	132	89	56	33	20	17	24
999.7	99	62	35	18	11	14	27
999.8	72	41	20	9	8	17	36
999.9	51	26	11	6	11	26	51
1000.0	36	17	8	9	20	41	72
1000.1	27	14	11	18	35	62	99
1000.2	24	17	20	33	56	89	132
1000.3	27	26	35	54	83	122	171
1000.4	36	41	56	81	116	161	216
1000.5	51	62	83	114	155	206	267
1000.6	72	89	116	153	200	257	324

The smallest value in this table is 6, corresponding to the coefficient of expansion .000020, and a length at 0° equal to 999.9 *mm.* The most probable assumption which we can make, therefore [without considering values intermediate between those given in the table], is that the length of the bar really was 999.9 *mm.* at 0° and that its coefficient of expansion was .000020.

The validity of this conclusion evidently depends upon the truth of the assumption that the bar has a constant coefficient of expansion.

The three observations given above indicate that the coefficient of expansion is greater from 100° to 200° than from 0° to 100°; but in the absence of a greater number of observations, there is no way of testing the truth of this indication. Any theory in regard to the variation of the coefficient of expansion would in general be investigated by the method of least squares.

It is of course unnecessary in practice to tabulate more than a few values, in order to see where the least square lies. This method of approximation is not necessarily longer than the calculus methods ordinarily employed; and has the advantage (which the teacher will see by referring to examples in well known text-books) of giving in many cases a more accurate result.

(*r*) *Advantage of the Arithmetic Mean.* If m is the arithmetic mean of n observed quantities, $o_1, o_2, \ldots o_n$, affected by like sources of error, and if the differences of these quantities from the mean are $d_1, d_2, \ldots d_n$, we have $o_1 = m + d_1$; $o_2 = m + d_2$; $\ldots$ $o_n = m + d_n$. Hence, adding and dividing by n, we have

$$m = (o_1 + o_2 + \ldots + o_n) \div n =$$
$$(m + d_1 + m + d_2 + \ldots + m + d_n) \div n$$
$$= m + (d_1 + d_2 + \ldots + d_n) \div n.$$

It follows that $d_1 + d_2 + \ldots + d_n = o$. The mean square of the differences from the mean m is

$$\epsilon^2 = (d_1^2 + d_2^2 + \ldots + d_n^2) \div n.$$

The mean square of the differences from any other value differing from m by the amount e is

$$\begin{aligned} E^2 &= [(d_1 + e)^2 + (d_2 + e)^2 + \ldots + (d_n + e)^2] \div n \\ &= [(d_1^2 + d_2^2 + \ldots + d_n^2) + (ne^2) + \\ &\quad (2d_1 e + 2d_2 e + \ldots + 2d_n e)] \div n \\ &= \epsilon^2 + e^2 + 2ne\,(d_1 + d_2 + \ldots + d_n) \div n. \end{aligned}$$

The last term in parenthesis is, as we have seen, equal to 0; hence we have simply

$$E^2 = \epsilon^2 + e^2.$$

That is, the mean square of the differences between the results of observation and their arithmetic mean is less than the mean square of the differences from any other value.

It follows from the principle of least squares that the arithmetic mean of a number of observations affected by like sources of error is the most probable value of the quantity observed which can be derived from these observations.

(*s*) *Weight of different results.* Let us suppose that on one day we have made 20 observations of the boiling-point of some alcohol, the result being 78°.58 ± .06; and that on another day we have made 80 observations in exactly the same manner, with the result 78°.48 ± .03. It follows from the last section that the most probable value of the boiling-point is that obtained by adding the total 100 results together, and dividing by their number (100). Let us suppose, however, that the original observations are lost. It is seen, nevertheless, that the sum of the first 20 must have been 20 × 78°.58, or 1571.60 and that the sum of the last 80 must have

been 80 × 78.48, or 6278.40. The total of the 100 observations must, therefore, have been 7850.00, hence the average is 78.50.

The number of observations of a given sort represented in a result determines what is called the weight of the result. Evidently if the weights of several data r_1, r_2, &c., are w_1, w_2, &c., the most probable value of the result, R, is

$$R = \frac{w_1 r_1 + w_2 r_2 + \&c.}{w_1 + w_2 + \&c.}.$$

Now let us suppose that the number of observations upon which different data depend is unknown. We have, for instance,

$$r_1 = 78^\circ.58 \pm .06$$
$$r_2 = 78^\circ.48 \pm .03;$$

it follows from the rules of probable error that the last result, having one half the probable error of the first, represents four times the number of observations, hence if we call $w_1 = 1$, $w_2 = 4$. This gives

$$R = \frac{78^\circ.58 + 4 \times 78^\circ.48}{4 + 1} = 78^\circ.50$$

as before. In the absence of any better indication of the relative weights of different results we may accordingly estimate these weights w_1, w_2, &c., from their probable errors p_1, p_2, &c., by means of the formulæ,

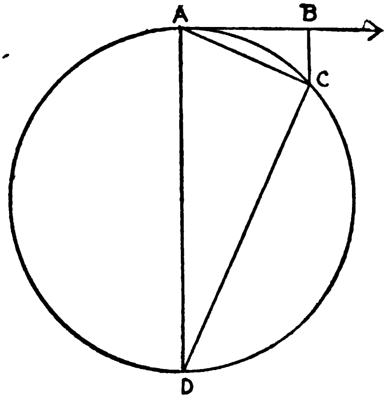
$$w_1 = 1 \div p_1^2;\ w_2 = 1 \div p_2^2,\ \&c. \qquad (2)$$

It is customary to extend this principle even to cases where results are known to be affected by unlike sources of error. The weight of a result is defined in general as the reciprocal of the square of its probable error.

APPENDIX XI.

PROOFS OF FORMULÆ.

(*a*) *Centrifugal Force.* When a body of mass m and velocity v in the direction AB is caused by a force f to move along the arc of a circle ACD, for a short time, t, it reaches a point C such that $AC = vt$. In the same time it is deflected in a direction at right angles to its original course through a distance BC, which by the law of falling bodies (§ 108) assuming the force to act constantly in the direction BC, is



$$BC = \tfrac{1}{2}\left(\frac{f}{m}\right)t^2.$$

the expression $(f \div m)$ taking the place of g. Drawing the diameter AD, and the chords AC and DC, we have by similar triangles

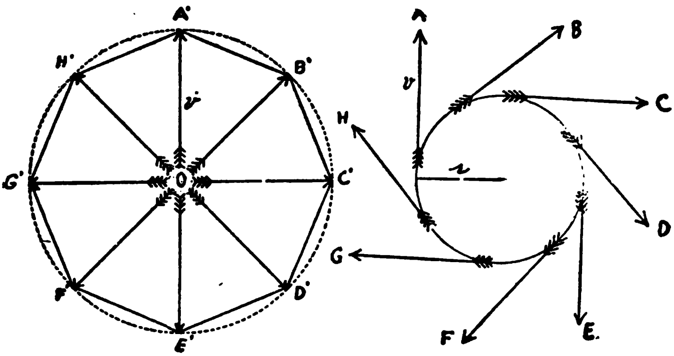
$$BC : AC :: AC : AD.$$

Substituting for BC its value from the formula, for AC its value (vt nearly), and for AD its value in terms of the radius ($2r$), we have

$$\tfrac{1}{2}\left(\frac{f}{m}\right)t^2 \times 2r = v^2t^2, \text{ or}$$

$$f = \frac{mv^2}{r}. \tag{1}$$

It is evident that the direction of the force is not constant, and that, as the chord AC is less than the arc, the former cannot be exactly equal to vt. We may, however, consider arcs so small that the change in direction of the force and the difference in proportion between the arc and its chord become less than any assignable quantity. The formula above is therefore exact.



A second demonstration of this formula depends upon the method of representing changes in velocity (§ 105). Let A, B, C, &c., be the velocities of the moving body at different points in a circle of radius r, and let A', B', C', &c., be the same when arranged so as to start from a common point, O. The difference in velocity between A' and B' is represented by the line $A'B'$; between B' and C', by $B'C'$, &c. Hence the perimeter of the figure $A'B'C' \ldots A'$ represents the total change of velocity in one complete revolution.

This method of representation would be exact if the velocity changed *abruptly* from A' to B', from B' to C', &c. In point of fact, it goes through every possible intermediate value. The real change of velocity is evidently equal to the perimeter of the circle $A'B'C' \ldots A'$, rather than that of the polygon. Let $2t$ be the time of one complete revolution; then since the radius of the circle in question is v, the total change in the time $2t$ is $2\pi v$, and the acceleration (a) or change of velocity per unit of time is

$$a = \frac{2\pi v}{2t} = \frac{\pi v}{t}.$$

At the same time the velocity, v, of a body making the circuit of a circle with radius, r, in the time, $2t$, is

$$v = \frac{2\pi r}{2t} = \frac{\pi r}{t}; \text{ hence } \frac{\pi}{t} = \frac{v}{r}.$$

Substituting this value, we find

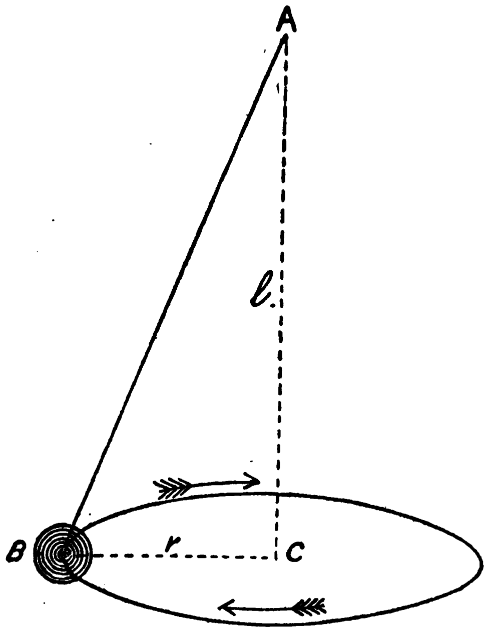
$$a = \left(\frac{\pi}{t}\right) v = \left(\frac{v}{r}\right) v = \frac{v^2}{r}.$$

The force (f) required to give this acceleration (a) to the mass (m) is according to the general definition (§ 12) equal to the product of the mass and acceleration; that is, as before

$$f = ma = \frac{mv^2}{r}.$$

The force f, which is exerted upon the body by some restraint, is evidently directed toward the centre of the circle in which the body is revolving, and is called accordingly a *centripetal force.* According to the general principle of action and reaction, an equal and opposite force is exerted by the body upon the restraint. This is called a *centrifugal force.*

(*b*) *Rotary Pendulum.* When a body of mass m, suspended by a cord AB (as in the figure), revolves about the centre, C, of a circle of radius, r, under a force of gravity equal to g dynes per gram, the centripetal force exerted by gravity is, by the principles of the composition and resolution



stated in § 105, equal to $mg \times \overline{BC} \div \overline{AC}$, or, calling $AC = l$, and $BC = r$,

$$f = \frac{mgr}{l}. \tag{1}$$

Now from the last section, if the time of a complete revolution is $2t$, the velocity v is

$$v = \frac{2\pi r}{2t} = \frac{\pi r}{t};$$

and the centripetal force is

$$f = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{\pi r}{t}\right)^2 = \frac{\pi^2 mr}{t^2}.$$

Equating the two values of f, we have

$$\frac{mgr}{l} = \frac{\pi^2 mr}{t^2},$$

or, cancelling m and r, and multiplying by l,

$$g = \frac{\pi^2 l}{t^2}, \qquad (2)$$

from which we find

$$t = \pi\sqrt{\frac{l}{g}} \qquad (3) \qquad \text{and} \quad l = \frac{gt^2}{\pi^2} \qquad (4)$$

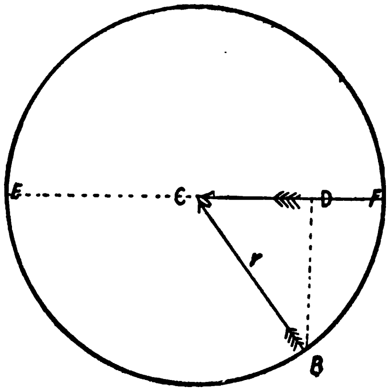
We note that the distance l is not the length of the pendulum, but its vertical component. When the displacement (r) is small, l may, however, be considered (practically) equal to the length of the pendulum.

(c) *Simple Pendulum.* The force f acting upon a rotary pendulum at any point B of its orbit is from the last section equal to $mg \times \overline{BC} \div \overline{AC}$. The component of this force in the direction of a given diameter $\overline{FE}$ is represented by the line $\overline{DC}$ in the figure. The distance passed over between the points F and B is equal to the arc $\overline{FB}$; the component of this distance in the direction of the diameter is $\overline{FD}$.

In the case of a simple pendulum, vibrating in the direction of the diameter $\overline{FE}$, the distance passed over is (neglecting the difference between a small arc and a straight line)

equal to $\overline{FD}$ (nearly). The horizontal component of the force urging it in the direction $\overline{DC}$ (again neglecting small differences owing to curvature in the path $\overline{FE}$) is equal to $mg \times \overline{DC} \div \overline{AC}$ (nearly).

Now let the simple and rotary pendula start together at F; since both are urged toward E with the same force, the components of velocity acquired in a given length of time will be the same. The projection D of the conical pendulum upon the diameter $\overline{FE}$ will therefore coincide with that of the simple pendulum. Starting at any point with the same component velocity, and urged forward by the same component forces, and having the same components of distance to traverse, the two pendula will evidently arrive at or opposite C at the same time, or allowing for curvature of the path $\overline{FC}$, at *nearly* the same point of time; and in the same way, both will arrive at E at (nearly) the same instant. In other words, the time of a simple pendulum is nearly the same as that of a rotary pendulum.



When the arc of oscillation is very small, we may consider the oscillations both of the simple and of the rotary pendulum to be confined to a given plane, hence we have as before

$$g = \frac{\pi^2 l}{t^2} \quad (1) \qquad t = \pi\sqrt{\frac{l}{g}} \quad (2) \text{ and } l = \frac{gt^2}{\pi^2}. \quad (3)$$

(*d*) *Compound Pendulum.* Multiplying both g and l in the last two equations by ml, we have

$$mlg = \frac{\pi^2 ml^2}{t^2}, \text{ and } t = \pi\sqrt{\frac{ml^2}{mlg}}. \qquad (1)$$

The quantity mlg, or $mg \times l$, is called the *directive force* (D) exerted by gravity upon the pendulum; the quantity ml^2 is called the moment of inertia (K) of the pendulum. Substituting these values, we have

$$D = \frac{\pi^2 K}{t^2} \quad (1) \qquad \text{or } t = \pi\sqrt{\frac{K}{D}}. \qquad (2)$$

When a pendulum of length l and mass m is deflected through a small distance r, the force brought to bear upon it is, as we have seen, equal to $mgr \div l$. This corresponds to a couple $(mgr \div l) \times l$, or $mlg \times \left(\frac{r}{l}\right)$. Substituting for the angle $(r \div l)$ its value in circular measure, a, we have for the couple c,

$$c = mlg \times a = D \times a.$$

The "directive force" (D) determines accordingly the couple which is brought to bear by gravity (g) when a mass (m) at the end of a lever-arm (l) is deflected through a given angle. The "moment of inertia" (ml^2) represents the couple necessary to give to a mass (m) at the end of a lever-arm (l) a unit angular velocity. This couple is evidently equal to the product of the mass (m) and the square of the lever-arm (l), because the force which must be exerted is the product of the mass (m) and the velocity (l) acquired; hence the couple, or product of the force (ml) and lever-arm (l), is equal to ml^2.

It is evident that if two bodies when deflected through a given angle are subjected to the same couple, and if the same couple produces the same angular acceleration, the time of oscillation must be the same. The formula above must apply, therefore, to compound as well as to simple pendula.

(*e*) *Reversible Pendula.* Let a pendulum of mass M, and moment of inertia $K = Mk^2$ about its centre of gravity, be suspended from a point at the distance x above the centre of gravity; then the directive force is

$$D = Mxg,$$

and the moment of inertia about the point of suspension is, as will be proved under (*j*).

$$K = Mk^2 + Mx^2,$$

where k represents the radius of gyration about the centre of gravity. The time of oscillation, t, is then

$$t = \pi\sqrt{\frac{K}{D}} = \pi\sqrt{\frac{Mk^2 + Mx^2}{Mxg}}.$$

Substituting $l = \dfrac{k^2 + x^2}{x}$, we have

$$t = \pi\sqrt{\frac{l}{g}},$$

where from the resemblance of the formula to that of a simple pendulum, l is called the length of an equivalent simple pendulum.[1]

Let us now suspend the pendulum at a distance y from the centre of gravity such that

[1] The length l is equal (by definition) to the distance between the centres of suspension and oscillation.

$$x + y = l, \text{ or } y = l - x = \frac{k^2 + x^2}{x} - x = \frac{k^2}{x}.$$

Then we have

$$t = \pi \sqrt{\frac{Mk^2 + My^2}{Myg}} = \pi \sqrt{\frac{Mk^2 + M\left(\frac{k^2}{x}\right)^2}{M\frac{k^2}{x}g}}$$

$$= \pi \sqrt{\frac{Mk^2x^2 + Mk^4}{Mk^2xg}} = \pi \sqrt{\frac{x^2 + k^2}{xg}} = \pi \sqrt{\frac{l}{g}}.$$

There are, accordingly, two distances, x and y, from the centre of gravity which give the same time of oscillation. We notice that $xy = k^2$, also that $x + y = l$. If, therefore, two points of suspension at the *unequal* distances x and y from the centre of gravity and on *opposite sides* of it are found to give the same time of oscillation, the length l of the equivalent simple pendulum may be found by measuring the distance between the points in question.

(*f*) *Errors of Adjustment in the Reversible Pendulum.* Let the pendulum of the last section be suspended from a point at a distance $x' = x + e$ from the centre of gravity. The corresponding value of y is

$$y' = \frac{k^2}{x'} = \frac{k^2}{x + e} = \frac{k^2}{x}\left(1 - \frac{e}{x}\right) \text{ nearly,}$$

if e is a small quantity. Hence the length (l') of the equivalent simple pendulum is

$$l' = x' + y' = x + e + y\left(1 - \frac{e}{x}\right) = x + y + \frac{ex - ey}{x}.$$

In the same way, if the pendulum be suspended from a point at a distance $y'' = y - e$ from the centre of gravity, the length of the equivalent simple pendulum is

$$l'' = x'' + y'' = x + y + \frac{ex - ey}{y}.$$

Calling the distance between the two points of suspension l, we have

$$l = x' + y'' = (x + e) + (y - e) = x + y;$$

hence we find

$$l' - l = x + y + \frac{ex - ey}{x} - (x + y) = \frac{ex - ey}{x}.$$

$$l'' - l = x + y + \frac{ex - ey}{y} - (x + y) = \frac{ex - ey}{y}, \text{ and}$$

$$\frac{l' - l}{l'' - l} = \frac{\frac{ex - ey}{x}}{\frac{ex - ey}{y}} = \frac{y}{x} = \frac{t' - t}{t'' - t} \text{ (nearly)},$$

assuming that the effects of small errors upon the time of oscillation are proportional to the effects upon the length of an equivalent simple pendulum.

We have, accordingly,

$$yt'' - yt = xt' - xt,$$

$$xt - yt = xt' - yt'' = (x - y)\, t' + yt' - yt''.$$

Hence finally,

$$t = t' + \frac{y}{x - y}\,(t' - t'').$$

We notice that

either $t'' > t' > t$, or $t'' < t' < t$;

in no case is t between t' and t''.

It may be observed that if x and y are equal, the expressions for l, l', and l'' become identical. In other words, the

pendulum does not respond under these conditions to a slight change in the points of suspension. To obtain accurate results, x should be several times greater (or less) than y.

The formulæ for the moment of inertia of a compound pendulum are established only for *parallel axes;* hence it is important that the axes passing through the two centres of suspension should be parallel. It is also important that the centre of gravity should lie in the plane of these two axes; if it does not, the dislocation should be allowed for in calculating the sum of the distances x and y.

(*g*) *Torsion Pendulum.* The moment of inertia of a (thin) ring about its axis is evidently equal to the mass M of the ring multiplied by the square of its mean radius R, for the whole mass of the ring is situated practically at the distance R from the axis (see (*k*) (1)). The directive force, D, of a wire which gives to a ring of mass M and mean radius R a time of oscillation, t, is therefore, according to section (*d*) formula (1):—

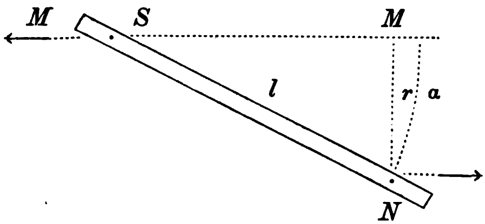
$$D = \frac{\pi^2 MR^2}{t^2} \tag{1}$$

The directive force, D, corresponds, as we have seen, to the couple required to produce a unit deflection in circular measure. The "coefficient of torsion" for 1° is evidently found by dividing the directive force D by the number of degrees in 1 circular unit of angle—that is 57°.29578, or 57°.3, nearly. We have, accordingly, in general

$$T = \frac{D}{57.3} = \frac{\pi^3 K}{180\, t^2} \tag{2}$$

(*h*) *Magnetic Pendulum.* When a magnet with poles of the strength $\pm s$, separated by a distance l, is suspended so that it is free to move only in a horizontal plane, we need

only to consider the forces exerted upon it by the horizontal component of the earth's magnetic field, which we will say is equal to H dynes per unit of magnetism. The two forces are accordingly $\pm s \times H$.



If the poles are relatively deflected east and west through a distance r, the couple tending to restore them to the magnetic meridian, MM, is $s \times H \times r$, or $(s \times l \times H) \times (r \div l)$. The ratio $r : l$ here determines (approximately) the small angle a in circular measure through which the magnet is deflected. The product $s \times l \times H$ corresponds accordingly to the product, $D = mlg$, in the case of a compound pendulum. Since the product of the strength (s) and distance (l) between the poles of a magnet determines its moment M, we have, substituting (see (d), (1)),

$$D = MH = \pi^2 \frac{K}{t^2},$$

where K is the moment of inertia (see k) and t the time of oscillation of the magnet.

(i) *Magnetometer.* Let a magnet with poles of the strength $\pm s$, separated by a distance l, be brought near a compass needle as in Fig. 200, ¶ 183. Let d be the mean distance of the poles from the needle. The nearer pole, being at a distance $d - \frac{1}{2} l$, will create a field of force f', such that

$$f' = \frac{\pm s}{(d - \frac{1}{2} l)^2}.$$

The further pole will create a field f'', such that

$$f'' = \frac{\mp s}{(d+\frac{1}{2}l)^2}.$$

The resultant field F is

$$F = f' + f'' = \pm s\left(\frac{1}{(d-\frac{1}{2}l)^2} - \frac{1}{(d+\frac{1}{2}l)^2}\right) =$$

$$\pm s\left(\frac{(d+\frac{1}{2}l)^2 - (d-\frac{1}{2}l)^2}{(d-\frac{1}{2}l)^2 \times (d+\frac{1}{2}l)^2}\right)$$

$$= \pm s\frac{d^2 + dl + \frac{1}{4}l^2 - (d^2 - dl + \frac{1}{4}l^2)}{d^4 - d^2l^2 + \frac{1}{4}l^4} = \frac{\pm 2sl}{d^3} \text{ nearly,}$$

neglecting l^2 in comparison with d^2.

Now if H is the horizontal component of the earth's field, and a the deflection, we have

$$F = H \tan a.$$

Equating the two values of F and substituting M for sl we have

$$H \tan a = \frac{2M}{d^3},$$

$$\text{or } \frac{M}{H} = \frac{1}{2}d^3 \tan a. \qquad (1)$$

We have already found in the last section,

$$MH = \frac{\pi^2 K}{t^2};$$

multiplying the values of $\frac{M}{H}$ and MH together, we find

$$\frac{M}{H} \times MH = M^2 = \frac{\pi^2}{2}\frac{Kd^3 \tan a}{t^2}$$

whence $$M = \frac{\pi}{t}\sqrt{\frac{1}{2}Kd^3 \tan a}.$$

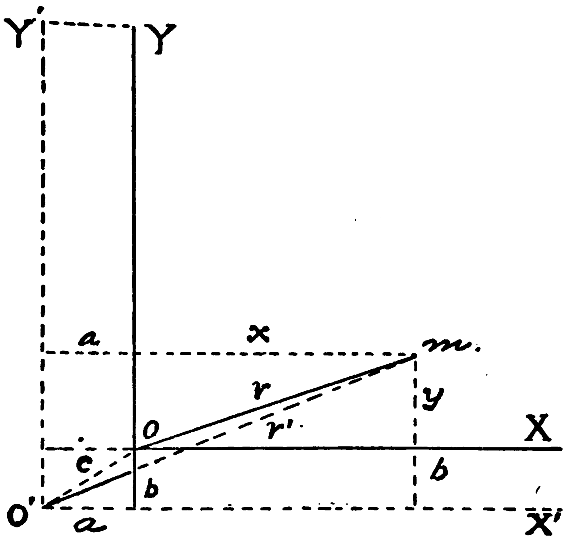
Dividing the value MH by that of $\frac{M}{H}$, we find

$$MH \div \frac{M}{H} = H^2 = \frac{2\pi^2 K}{t^2 d^3 \tan \alpha},$$

whence
$$H = \frac{\pi}{t}\sqrt{\frac{2K}{d^3 \tan \alpha}}. \qquad (2)$$

Instead of d^3 in these formulæ, we should strictly substitute $d^3 - dl^2 + \frac{1}{4}l^4 \div d$. The last term may almost always be neglected.

(j) *Moments of Inertia about Parallel Axes.* The moment of inertia (K) of a body about a given axis may be defined as the sum of the moments of inertia of the separate



masses of which it is composed. That is, denoting a given mass by m, and its distance from the axis by r,

$$K = \Sigma\, mr^2; \qquad (1)$$

or, if we consider the total mass, M, of the body as divided into small *equal* masses, m,

$$K = M\overline{r^2} = Mk^2, \tag{2}$$

where k is the "radius of gyration" about the axis O (or the radius of the mean square). Denoting by x and y the distance of m from two rectangular planes passing through O, we have, substituting

$$K = M\overline{r^2} = M\overline{(x^2 + y^2)} = M\,(\overline{x^2} + \overline{y^2}).$$

In the same way, the moment of inertia about a parallel axis O', at a distance $OO' = c$ from O, is determined by the distances $x + a$ and $y + b$ from two planes $O'X'$ and $O'Y'$ at distance b and a from OX and OY, so that $a^2 + b^2 = c^2$. We have accordingly

$$K' = M\overline{(r')^2} = M\overline{(x')^2 + (y')^2} = M\overline{(x + a)^2 + (y + b)^2}$$

$$= M\,\overline{x^2 + 2ax + a^2 + y^2 + 2by + b^2}$$

$$= M\,(\overline{x^2} + \overline{y^2} + \overline{2ax} + \overline{2by} + \overline{a^2} + \overline{b^2}).$$

Now if O contains the centre of gravity, we have by definition $\overline{x} = 0$ and $\overline{y} = 0$, hence $\overline{2ax} = 0$ and $\overline{2by} = 0$; this gives

$$K' = M\,(\overline{x^2} + \overline{y^2} + a^2 + b^2) = M\overline{r^2} + Mc^2 =$$

$$Mk^2 + Mc^2 = K + Mc^2 \tag{3}$$

It follows that the moment of inertia (K) of a body of mass M about an axis passing through the centre of gravity is less than that (K') about any other parallel axis at the distance c by the amount Mc^2, which is equal to the moment of inertia of the whole mass M concentrated at a single point at the distance c from the axis.

The quantity Mc^2 represents the difficulty of causing the centre of gravity of a body to begin to rotate in an arc of radius c. The quantity $M\overline{r^2}$ or Mk^2 represents the difficulty of making a body begin to rotate about its own centre of gravity. When a body begins to rotate about a given axis and also about its centre of gravity at the same time, both sources of difficulty are met.

(*k*) *Calculation of Moments of Inertia.* The moment of inertia of a small mass m, at a distance l from its axis of revolution, is by definition ml^2. The moment of inertia of a thin ring of mass M and mean radius R about its axis is accordingly MR^2, for the ring may be thought of as composed of n small masses of the magnitude m, and such that $nm = M$, each of the masses being situated at a distance R from the axis of revolution. Since the moment of inertia for each mass is mR^2, the total is $n \times mR^2$ or $(nm)\ R^2$ or MR^2. Hence we have

$$K = MR^2. \tag{1}$$

The moment of inertia of a long thin bar of length l and mass M is evidently less than $M \times (\frac{1}{2}L)^2$, because the whole of the weight is not situated at the end of the bar. The moment of inertia increases in fact according to the square of the distance x measured from the centre of the bar outward. Hence, applying the method of averages, we find for either end of the bar

$$\frac{K}{2} = \frac{M}{2} \quad 0 \underset{x^2}{\overline{\qquad}} \tfrac{1}{2} = \tfrac{1}{3}\,\frac{M}{2}\left(\frac{L}{2}\right)^2,$$

or

$$K = \tfrac{1}{12} ML^2 \tag{2}$$

The moment of inertia of such a bar about an axis at a distance a from its middle point is, according to the last section,

$$K = \tfrac{1}{12}ML^2 + Ma^2. \qquad (3)$$

A bar of length L and breadth B may be considered as composed of a series of thin parallel bars of length l, each having a moment of inertia $\tfrac{1}{12}ML^2 + My^2$, depending upon its distance y from the axis. Imagining such a bar to be divided longitudinally into halves by a plane passing through the axis, we find, averaging for either half,

$$\frac{K}{2} = \overset{0 \qquad\qquad \frac{B}{2}}{\overline{\tfrac{1}{12}ML^2 + My^2}} = \overset{0 \qquad \frac{B}{2}}{\overline{\tfrac{1}{12}ML^2}} + \overset{0 \quad \frac{B}{2}}{\overline{My^2}}$$

$$= \tfrac{1}{12}ML^2 + \tfrac{1}{3}M\left(\frac{B}{2}\right)^2,$$

whence
$$K = \tfrac{1}{12}M(L^2 + B^2). \qquad (4)$$

The moment of inertia of a thin ring of radius r about one of its diameters as an axis is found by averaging the square of the distance $(r \sin x)^2$ of points subtending all possible angles x from the centre of the ring, and multiplying by the mass M of the ring. Since $\overline{\sin^2 x} = \frac{1}{2}$, (see IX. ($j$)), we find simply

$$K = \tfrac{1}{2}MR^2. \qquad (5)$$

A thin disc of mass M and radius R can be regarded as a series of rings with increasing radius and mass. The mass of a ring of radius x bears to one of radius R (of the same breadth, thickness and density) the ratio $x : R$; and since the moments of inertia are proportional to the masses and to the squares of the radii, they are to each other in the ratio $x^3 : R^3$; hence on the average the moment of inertia of a series of rings occupying a total breadth from 0 to R is to that of a series of the same breadth, all having the radius R, in the proportion (see IX. (h)),

$$ {}^{0}\overline{x^3 \div R^3}^{\,r} = \tfrac{1}{4}. \qquad (6)$$

The area covered by a series of rings of the breadth R and radius R would, however, be $2\pi R \times R = 2\pi R^2$; while the area actually covered by the rings is πR^2; hence (assuming that masses and areas are proportional) the mass with which we have compared the disc is $2M$. We conclude that the moment of inertia of the disc about its axis is

$$K = \tfrac{1}{4} 2MR^2 = \tfrac{1}{2}MR^2; \qquad (7)$$

and for the moment of inertia of a disc about a diameter,

$$K = \tfrac{1}{4} 2\frac{M}{2} R^2 = \tfrac{1}{4}MR^2. \qquad (8)$$

The moment of inertia of a disc of mass M about a diameter can also be found by averaging moments of inertia corresponding to a given distance x from the diameter. The moment of inertia of the disc about an axis parallel to the diameter and passing through the axis of the disc at a distance L from it is, according to the last section,

$$K = \tfrac{1}{4}MR^2 + ML^2. \qquad (9)$$

A cylinder with a transverse axis passing through its middle point may be regarded as a series of discs situated at regularly increasing distances x from the axis. The moment of inertia of any such disc is

$$K = \tfrac{1}{4}MR^2 + Mx^2;$$

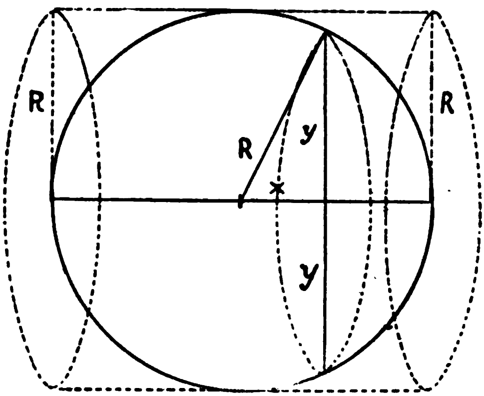
hence, averaging (see IX. (f)), we find for either half of the cylinder,

$$\frac{K}{2} = {}^{0}\overline{\tfrac{1}{4}\frac{M}{2}R^2 + \frac{M}{2}x^2}^{\,\frac{L}{2}} = {}^{0}\overline{\tfrac{1}{4}\frac{M}{2}R^2}^{\,\frac{L}{2}} + {}^{0}\overline{\frac{M}{2}x^2}^{\,\frac{L}{2}}$$

$$= \tfrac{1}{4}\frac{M}{2}R^2 + \tfrac{1}{3}\frac{M}{2}\left(\frac{L}{2}\right)^2;$$

whence $$K = \tfrac{1}{12}ML^2 + \tfrac{1}{4}MR^2. \qquad (10)$$

A sphere of radius R may be regarded as a series of discs of varying weight and radius. The weight of a disc at the distance x from the centre of the sphere bears to that of one



of the same thickness at the centre the ratio $y^2 : R^2$: the moments of inertia are proportional to the weight multiplied by the square of the radii, hence are to each other as $y^4 : R^4$. The moment of inertia of a sphere compared with that of a cylinder of the same length and diameter is therefore (see IX. (i)),

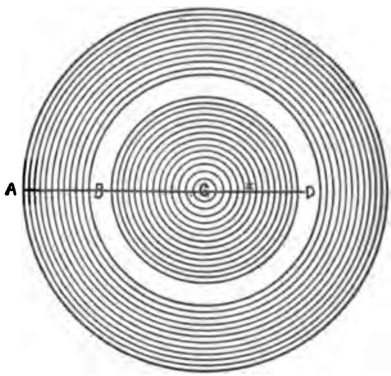
$$\overset{-R\;\overline{\qquad\quad}\;+R}{\frac{y^4}{R^4}} = \overset{-R\;\overline{\qquad\qquad\qquad}\;+R}{\frac{(R^2 - x^2)^2}{R^4}} = \overset{-R\;\overline{\qquad\qquad\qquad\qquad}\;+R}{1 - 2\frac{x^2}{R^2} + x^4}$$

$$= 1 - \tfrac{2}{3} + \tfrac{1}{5} = \tfrac{8}{15}. \qquad (11)$$

The volume of the sphere is $\frac{4\pi}{3} R^3$; that of the cylinder is $2R \times \pi R^2 = 2\pi R^3$; that is, $1\frac{1}{2}$ times as great as the sphere. Hence if the mass of the sphere is M, that of the cylinder is $1\frac{1}{2} M$, and its moment of inertia about its axis is $\frac{1}{2} \times 1\frac{1}{2} M \times R^2 = \frac{3}{4} MR^2$. The moment of inertia of the sphere, being $\frac{8}{15}$ that of the cylinder, is accordingly,

$$K = \frac{8}{15} \times \frac{3}{4} MR^2 = \frac{2}{5} MR^2 \qquad (12)$$

(*l*) *Coefficient of Viscosity.* Let a tube of radius r and length l be filled with a liquid of which all is frozen except a tubular section of (mean) radius x, and unit thickness. Let p be the difference of pressure at the two ends of the tube, then the force on the core is $p \times \pi x^2$ (nearly). This is resisted by a force equal to the velocity v of the core multiplied by the (mean) area of the opposite surfaces of the tubular section, $2\pi x l$, and multiplied by the coefficient of viscosity, η. That is,—



$$2\pi x \eta v l = p \pi x^2, \text{ whence}$$

$$v = \frac{px}{2\eta l}.$$

The quantity (or volume) q of (principally frozen) liquid which flows through the tube in the time t is

$$q = \pi x^2 v t; \text{ hence, substituting,}$$

$$q = \pi x^2 vt = \frac{\pi x^2 pxt}{2\eta l} = \frac{\pi pt}{2\eta l}\, x^3.$$

If now a new tubular section be melted, either inside or outside of the former section, the relative velocity of all points within the new section will be increased as much as if the former section were solid, hence the *increase* in the flow will be represented by the same formula as before. We may suppose all the sections to be melted one by one, each contributing a certain amount to the flow. If Q is the total flow, $Q \div r$ must be the *average* flow for each section; hence we have (see IX. (h)),

$$\frac{Q}{r} = \frac{\pi pt}{2\eta l} \cdot \overset{o\text{———}r}{x^3} = \frac{\pi pt}{2\eta l} \cdot \tfrac{1}{4} x^3 = \frac{\pi ptr^3}{2\eta l}.$$

Now if the pressure p is due to a hydrostatic column of height h and density d, and if the acceleration of gravity is g, we have

$$p = ghd.$$

The weight of liquid delivered is, moreover, $Q \times d$, so that

$$Q = \frac{w}{d}.$$

Making these substitutions, we have, solving for η,

$$\eta = \frac{\pi g d^2 h r^4 t}{8wl}.$$

(m) *Coefficients of Elasticity.*[1] When a unit cube is subjected on all sides to a pressure P, its volume is reduced by an amount v, given by the equation

$$v = \frac{P}{M},$$

[1] The formulæ here derived apply only to "isotropic" substances.

where M is by definition the "coefficient of resilience of volume." The increase (l, b, and t,) of the length, breadth, and thickness, are of course each equal to $\frac{1}{3}v$. That is,

$$l = b = t = \tfrac{1}{3}\frac{P}{M}.$$

When a unit cube is subjected to a pressure P on two opposite faces so as to diminish its length, and an equal tension on two other faces so as to increase its breadth, its volume remains the same (nearly); but the length and breadth are altered so that

$$\frac{1+b}{1-l} - 1 = \frac{P}{S},$$

where S is by definition the "coefficient of simple rigidity."

Since b and l are small and nearly equal, we have

$$\frac{1+b}{1-l} - 1 = 1 + b + l - 1 \text{ (nearly)} = 2b = 2l = \frac{P}{S}.$$

whence $$b = l = \tfrac{1}{2}\frac{P}{S}.$$

When a unit cube is subjected simply to a pressure P in the direction of its length, the latter becomes shortened by an amount l, such that

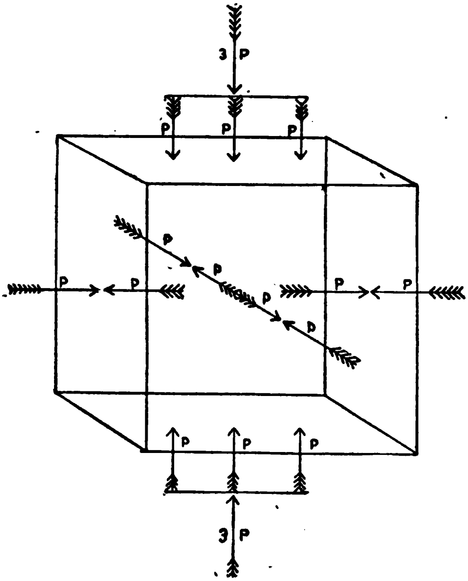
$$l = \frac{P}{Y},$$

where Y is by definition "Young's modulus of elasticity." At the same time the breadth and thickness increase by equal amounts, b and t,

$$b = t = \mu l = \mu\frac{P}{Y},$$

where μ is by definition "Poisson's ratio."

Now let a unit cube be subjected to the pressure $3P$ in the direction of its length; and let equal and opposite pressures $\pm P$ be applied to each of its sides. We shall then have a uniform pressure, P, on each surface, tending to diminish the volume, combined with two pairs of equal and opposite pressures, P, one pair tending to increase the breadth, the other pair tending to increase the thickness, and both pairs tending



to diminish the length. Hence we have, adding the three effects upon the length together,

$$l' = \tfrac{1}{3}\frac{P}{M} + \tfrac{1}{2}\frac{P}{S} + \tfrac{1}{2}\frac{P}{S} = \tfrac{1}{3}\frac{P}{M} + \frac{P}{S}.$$

We have also, remembering that the total longitudinal pressure is $3P$,

$$l' = 3 \times \frac{P}{Y};$$

hence, equating the two values of l', and dividing through by $3P$,

$$\frac{1}{Y} = \frac{1}{9M} + \frac{1}{3S}. \qquad (1)$$

By means of this equation either one of the coefficients Y, M, or S can be found if the other two are given. If S and M are proportional to the numbers 6 and 10, for instance, Y is represented by the number 15.

The increase of breadth and thickness can be found in the same way as the length, remembering that only one pair of equal and opposite forces tends to increase each, and that the effects of compression tend to diminish the result. We have

$$b' = t' = \tfrac{1}{2}\frac{P}{S} - \tfrac{1}{3}\frac{P}{M}.$$

Dividing b' (or t') by l', we find

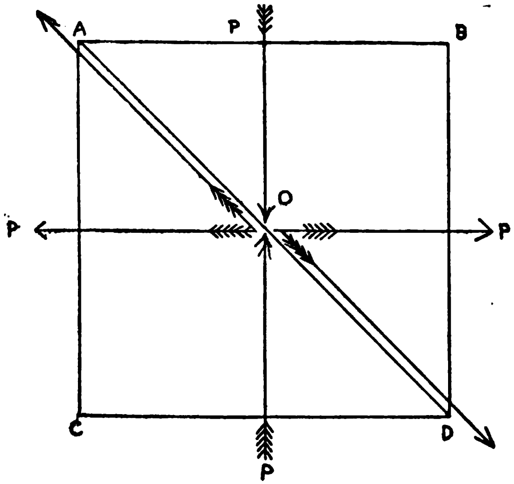
$$\mu = \frac{b'}{l'} = \frac{t'}{l'} = \frac{\frac{1}{2}\frac{P}{S} - \frac{1}{3}\frac{P}{M}}{\frac{1}{3}\frac{P}{M} + \frac{P}{S}} = \frac{3M - 2S}{6M + 2S}. \qquad (2)$$

It is seen that if S and M are proportional to the numbers 6 and 10, $\mu = \frac{1}{4}$.

(n) *Shearing Stresses and Strains.* Let a unit cube be acted upon by two equal and opposite pressures, P, tending to reduce its length, and by two equal and opposite tensions, also equal to P, tending to increase its breadth. The resultants may evidently be represented by two forces, OD . . . and OA, . . . each equal to $\sqrt{2} \times P$. These tend to make the two halves of the cube slide relatively in the directions AD

and DA. This tendency is resisted by the plane AD, the area of which is $\sqrt{2}$. Hence the intensity of the tangential or "shearing" stress is $\sqrt{2} \times P \div \sqrt{2} = P$.

We have seen that if S is the simple rigidity of a body subjected to a pair of stresses at right angles equal to $\pm P$,



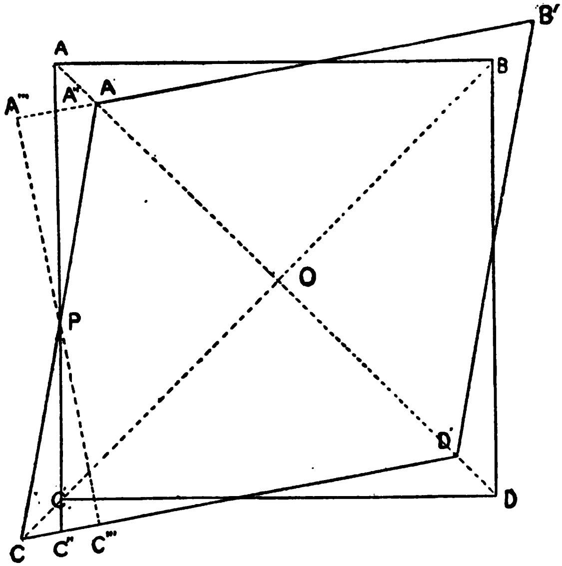
the dimensions of a unit of length become altered by the amount

$$l = \pm \tfrac{1}{2} \frac{P}{S}.$$

If, therefore, the stresses are exerted along the *diagonals* of the cube (AOD and BOC) one of these diagonals, AOD for instance, will be shortened, the other lengthened by the amount $\sqrt{2} \times l$ (since the change of length is proportional to the distance affected). It follows that AA', DD', &c., being equal to one half the change of length in each case, are all equal to $\tfrac{1}{2} \sqrt{2} \times l$, or

$$AA' = \tfrac{1}{2}\sqrt{\tfrac{1}{2}}\,\frac{P}{S}.$$

From P, where AC and $A'C'$ intersect, draw a perpendicular PA''' to $B'A'$ and PC''' to $D'C'$; and let AC cut $B'A'$



and $D'C'$ at A'' and C''. Then, by construction, in the (nearly) isosceles right-angled triangle $AA''A'$,

$$A'A'' = \sqrt{\tfrac{1}{2}} \times AA' = \sqrt{\tfrac{1}{2}} \times \tfrac{1}{2}\sqrt{\tfrac{1}{2}}\,\frac{P}{S} = \tfrac{1}{4}\frac{P}{S} \text{ (nearly).}$$

We have also, by construction,

$$A'A'' = A''A''' = C'C'' = C''C''' \text{ (nearly)}, = \tfrac{1}{4}d,$$

where d is the total dislocation of the side $A'B'$ with respect to $C'D'$, which, since the distance between the sides is 1, is equal to the tangental or "shearing" strain. Hence we have

$$d = 4AA' = \frac{P}{S} \text{ (nearly).}$$

or

$$S = \frac{P}{d}.$$

We have seen that P represents the shearing stress, d the shearing strain; S is the "simple rigidity." It might also be called the "modulus of shearing."

The constant S in formulæ involving transverse stresses and strains evidently takes the place of Young's modulus in formulæ where these stresses and strains are longitudinal.

(*o*) *Coefficients of Torsion.* In a thin tube of length l, thickness t, and mean radius r, the cross-section is $2\pi rt$, and if the angle of torsion is α in circular measure, the twist per unit of length is $\alpha \div l$, so that two points at the unit distance (measured longitudinally) are dislocated through the distance $r \times \alpha \div l$. The force necessary to produce such a dislocation between surfaces of the area $2\pi rt$ is

$$f = 2\pi rt \times r \times \alpha \div l \times S,$$

where S is the "coefficient of simple rigidity." The couple required is accordingly

$$c = f \times r = 2\pi Sr^3 t\alpha \div l.$$

The "directive force" (d), or ratio of the couple to the angle of torsion in circular measure is

$$d = \frac{c}{\alpha} = \frac{2\pi Sr^3 t}{l}.$$

A cylindrical rod (or wire) may be considered as a series of tubes with radii varying from 0 to r. The directive force for each tube is less than that of a tube with the radius r in the proportion $x^3 : r^3$; hence the directive force of a rod

is less than that of a tube with a radius and thickness equal to the radius of the rod in the proportion (see IX. (*h*)),

$$\frac{0 \text{———} r}{x^2 \div r^3} = \tfrac{1}{2}.$$

Now the directive force of tube of radius r and thickness r would be

$$d = \frac{2\pi S r^4}{l};$$

hence that of the rod is (see section (*d*) formula (1));

$$D = \tfrac{1}{4} \cdot \frac{2\pi S r^4}{l} = \frac{\pi}{2} \cdot \frac{S r^4}{l}. \qquad (1)$$

Dividing the directive force by $\frac{360}{2\pi}$ (the number of degrees in 1 unit of angle), we find the coefficient of torsion per degree,

$$T = \frac{2\pi D}{360} = \frac{\pi^2}{360} \cdot \frac{S r^4}{l}. \qquad (2)$$

Given D or T, the coefficient of simple rigidity, S, may evidently be found by the formula

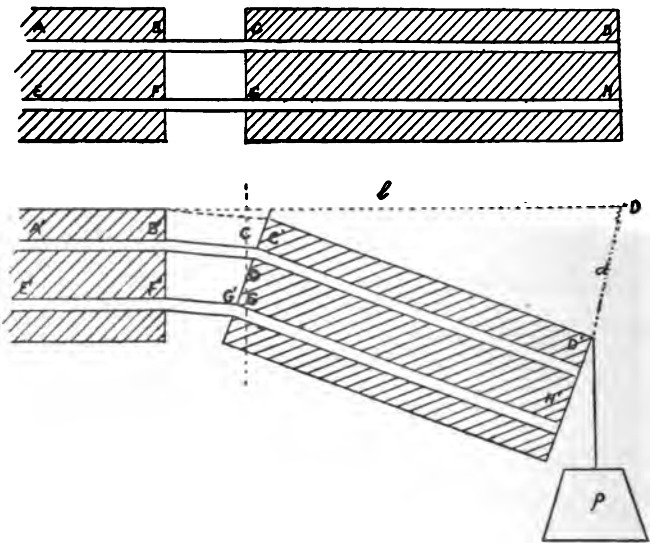
$$S = \frac{2lD}{\pi r^4} = \frac{360 l T}{\pi^2 r^4}. \qquad (3)$$

If the directive force D is determined by the time of oscillation t of a body of moment of inertia K, we have, substituting the value of D, namely $\pi^2 K \div t^2$ (see section (*d*) formula (1)),

$$S = \frac{2\pi l K}{r^4 t^2}. \qquad (4)$$

(*p*) *Transverse Elasticity.* Let a beam consisting of two thin rods, AD and EH, of length L, breadth B, and unit thickness, be bound in some light inelastic material except-

ing the unit lengths BC and FG, which are at a distance l from the ends of the rod, and at a mean distance t from each other; and let a transverse force p applied at the end of the rod produce a deflection $\overline{DD'} = d$.



Drawing COG parallel to $B'F'$ and bisecting $C'O'G'$, so that $OC' = OG' = \frac{1}{2}\, t$, we have from (nearly) similar triangles,

$$CC' : OC' :: GG' : OG' :: d : l,$$

or $$CC' = GG' = \tfrac{1}{2}td \div l.$$

The forces brought into play by stretching the rods BC and FG of unit length and with the cross-section B are, if Young's modulus is Y,

$$f = \pm\, \overline{CC'} \times BY = \pm\, \tfrac{1}{2}tBYd \div l,$$

and the couple produced is

$$C = f \times t = (\tfrac{1}{2}tBYd \div l) \times t = \tfrac{1}{2}t^2BYd \div l.$$

This couple must be equal to that due to the force p on the arm l, hence

$$p \times l = \tfrac{1}{2}t^2BYd \div l, \text{ whence}$$

$$Y = \frac{2pl^2}{t^2Bd}.$$

It would be possible to find Young's modulus by this formula (remembering that the result is to be multiplied by the length BC and divided by the thickness of the rods, if these are not unity); in practice we employ, however, a solid rod, of thickness T, which we may consider as composed of a series of pairs of rods of the unit thickness, equal in number to $\frac{1}{2}T$. If the total couple produced by these rods is C, the average couple is evidently $C \div \frac{1}{2}T$, or $2C \div T$. Hence we have (see IX. (f)),

$$\frac{2C}{T} = \overset{0\,\rule{3em}{0.4pt}\,T}{\tfrac{1}{2}\frac{t^2BYd}{l}} = \tfrac{1}{2}\cdot\tfrac{1}{3}\,\frac{T^2BYd}{l} = \tfrac{1}{6}\,\frac{T^2BYd}{l},$$

from which,

$$d = \frac{12Cl}{BT^3Y} = \frac{12\,(F \times l)\,l}{BT^3Y} = \frac{12Fl^2}{BT^3Y},$$

where F is the force producing the couple C. Now suppose that the rod is released from its restraint to a distance L from the free end, each portion contributing an amount d to the total deflection D, due to the bending of all the portions. The average deflection due to each unit of length of the rod being $D \div L$, we have

$$\frac{D}{L} = \frac{12Fl^2 \Big|_0^L}{BT^3Y} = \frac{12 \times \frac{1}{3}L^2}{BT^3Y} = \frac{4FL^2}{BT^3Y},$$

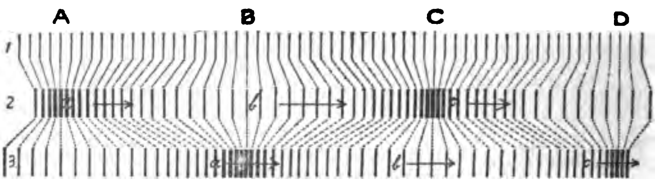
from which we find

$$Y = \frac{4FL^3}{BT^3D}.$$

When a rod of length l, breadth b, and thickness t, supported at both ends and loaded in the middle by a force f, is deflected through a distance d, each support reacts with a force $F = \frac{1}{2}f$, and since the middle of the rod remains horizontal, the length bent is $L = \frac{1}{2}l$. Substituting these values we find

$$Y = \frac{4 \cdot \frac{1}{2}f(\frac{1}{2}l)^3}{bt^3d} = \frac{fl^3}{4bt^3d}$$

(*q*) *Longitudinal Wave Motion.* The strata of a medium which in a state of rest would be equally spaced, as in the series (1), in the figure, are when transmitting a wave of



sound, crowded together is some places, as A 2, C2, B3, and D3, and more or less separated in others. It is seen that a comparatively small distance traversed by the strata between (2) and (3) accounts for the apparent movement of the condensation from A to B. We will suppose this apparent movement to continue indefinitely with the velocity v, and that several imaginary points, a, b, c, &c., move with the same ve-

locity and in the same direction, so that one of them, a for instance, is always in the denser portion of the "wave," while another point, b, is in a comparatively rarefied portion. The number of strata traversed by a in a given length of time must be approximately the same as that traversed by b, for if a left many more strata behind it than b, the strata would soon become exhausted from between them, and if b left more behind it than a, there would be an indefinite condensation of strata. Both of these suppositions are contrary to the conditions which we have assumed. Now if n' is the number of strata per unit of distance at a, n'' the number at b, v the velocity of the points a and b, v' that of the strata at a, and v'' that of the strata at b, the relative velocities are $v - v'$ and $v - v''$ respectively; the number passed by a in the time t is $(v - v')\, n't$; and that passed by b is $(v - v'')\, n''t$; hence

$$(v - v')\, n't = (v - v'')\, n''t.$$

Now the densities of the medium, d' at a, and d'' at b, are evidently proportional to the number of strata per unit of distance, hence

$$\frac{v - v'}{v - v''} = \frac{n''}{n'} = \frac{d''}{d'},$$

from which we find

$$\frac{v - v'}{v - v''} - 1 = \frac{d''}{d'} - 1 = \frac{v'' - v'}{v - v'} = \frac{d'' - d'}{d'},$$

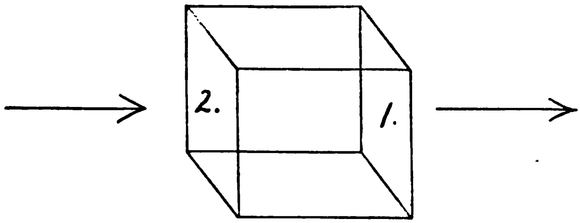
or
$$\frac{v'' - v'}{d'' - d'} = \frac{v - v'}{d'}.$$

Now if d' represents the mean density, d, of the medium and v' the corresponding velocity, which, in the absence of any motion of translation [e.g. wind] we will assume to be 0, we have, substituting,

$$\frac{v''-0}{d''-d}=\frac{v-0}{d}, \text{ or } \frac{v''}{v}=\frac{d''-d}{d}. \qquad (1)$$

That is, the velocity of a given particle is to the velocity of the wave as the difference in density from the mean density is to the mean density of the medium. We notice that in the denser portions of a wave the particles are moving with it; but in the rarer portions they are moving against it. Under these conditions only is longitudinal wave-motion possible.

(*r*) *Velocity of Sound in Air.* Let 1 and 2 in the figure be two points on opposite faces of a centimetre cube of air,



where the pressures are p_1 and p_2, the densities d_1 and d_2, the velocities of the particles v_1 and v_2, respectively. Then if a wave of sound moves from 2 to 1 with the velocity v, it will occupy a time t in traversing the (unit) distance in question such that

$$t=\frac{1}{v}.$$

The forces acting upon the two faces of the cube p_2 and p_1, being opposite in direction, have a resultant f in the direction of the wave motion,

$$f=p_2-p_1.$$

The mass acted upon is numerically equal to the density of the air,

$$m = d \text{ (nearly)}.$$

The velocity acquired is equal to the difference between the original velocity, v_1 at the point 1 and the final velocity, v_2, which with the other properties of the point 2 are carried to the point 1 by the progression of the wave. We have, therefore,

$$\Delta v = v_2 - v_1.$$

Substituting the values of t, f, m, and Δv, in the formula expressing the general law of motion,

$$f \times t = m \times \Delta v, \text{ we have}$$

$$(p - p_1) \times \frac{1}{v} = d \times (v_2 - v_1).$$

Substituting for v_2 and v_1 their values from the last section, namely

$$v_2 = \frac{v}{d}(d_2 - d) \text{ and } v_1 = \frac{v}{d}(d_1 - d) \text{ we have}$$

$$(p_2 - p_1) \times \frac{1}{v} = d \times \left(\frac{v}{d}(d_2 - d) - \frac{v}{d}(d_1 - d)\right)$$

$$= v\,(d_2 - d_1).$$

Hence we find

$$v^2 = \frac{p_2 - p_1}{d_2 - d_1}. \qquad (1)$$

If air *suddenly compressed* obeyed the law of Boyle and Mariotte (as Newton wrongly supposed), we should have

$$P : D :: p_2 : d_2 :: p_1 : d_1 :: p_2 - p_1 :: d_2 - d_1, \text{ \&c.}$$

In fact, however, so much heat is developed by sudden compression that the increase of pressure is, in the case of air,

about 1.408 times, and in general κ times as great as it would be according to the law of Boyle and Mariotte. We have accordingly,

$$v^2 = \frac{p_2 - p}{d_2 - d} = \kappa \frac{P}{D}. \qquad (2)$$

Substituting for κP the symbol E, representing the "coefficient of volume resilience" we have finally,

$$v = \sqrt{\frac{E}{D}}. \qquad (3)$$

This formula applies to the velocity of sound in any medium, provided that E represents that modulus of elasticity which resists the dislocation of strata accompanying the propagation of the sound.

In the case of a thin wire, we substitute for E "Young's modulus of elasticity," if the vibrations are longitudinal, or the "simple rigidity" if the vibrations are torsional.

(*s*) *Index of Refraction of a Prism.* When a ray of light, $FGHI$, passes through an equilateral prism, AJL, in a direction GH, parallel to the base, JL, the angles KGH and KHG between the ray and the normals BK and CK, are evidently each equal to the angle of refraction r. The sum of these angles ($2r$) is the supplement of BKC; and the prism angle A is also the supplement of BKC; hence

$$r = \tfrac{1}{2} A. \qquad (1)$$

From the equality of the angles KGH and KHG within the prism, follows that of the angles BGF and CHI outside of the prism; these are accordingly each equal to the angle of incidence, i. Now the ray of light is deviated at the point G through an angle DGF, and at H through an equal angle; hence the total angle of deviation,

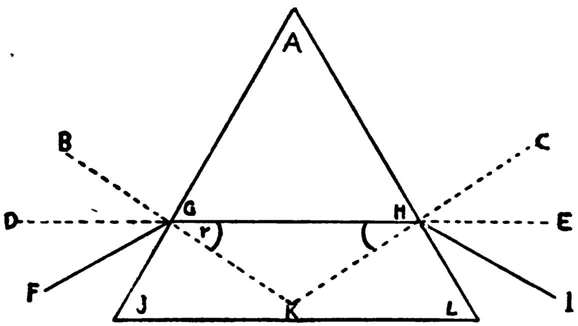
$$d = 2\,DGF = 2(BGF - BGD) = 2(BGF - KGH)$$
$$= 2\,(i - r).$$

Hence

$$i - r = \tfrac{1}{2}d, \text{ or } i = r + \tfrac{1}{2}d = \tfrac{1}{2}A + \tfrac{1}{2}d. \qquad (2)$$

Substituting these values of i and r in the formula § 102, we have

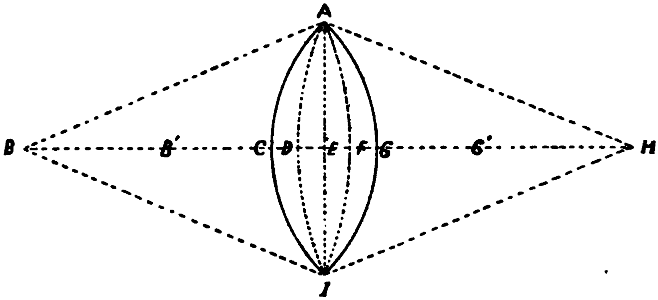
$$\mu = \frac{\sin\,(\frac{1}{2}A + \frac{1}{2}d)}{\sin\,(\frac{1}{2}A)}. \qquad (3)$$



(*t*) *Index of Refraction of a Lens.* When waves of light from a point B are brought to a focus at H, it is evident that in a given length of time different distances are traversed by different portions of the wave. Drawing the arcs AFI and ADI with B and H as centres, also the straight line AEI, we see that the path BAH is longer than BEH by the amount DF. In the same time that light traverses a distance CG through the lens it passes accordingly through a distance $CG + DF$ in air. The index of refraction is, accordingly,

$$\mu = \frac{CG + DF}{CG}.$$

The object of the present investigation is simply to express DF and CG in terms of the radii of curvature $B'G$ and $G'C$) and focal lengths (BE and HE) of the lens. We have by geometry



$$(EF) = (AE)^2 \div (BE) \text{ and } (DE) = (AE)^2 \div (HE).$$

Hence

$$DF = DE + EF = (AE)^2 \times (1 \div f_1 + 1 \div f_2),$$

where f_1 and f_2 represent the conjugate focal lengths.

We have similarly,

$$(CG) = (AE)^2 \times (1 \div B'E + 1 \div G'E) =$$

$$(AE)^2 \times (1 \div R_1 + 1 \div R_2) \text{ nearly,}$$

neglecting the relatively small distances CE and EG in comparison with the radii R_1 and R_2. Substituting these values and cancelling $(AE)^2$ we find

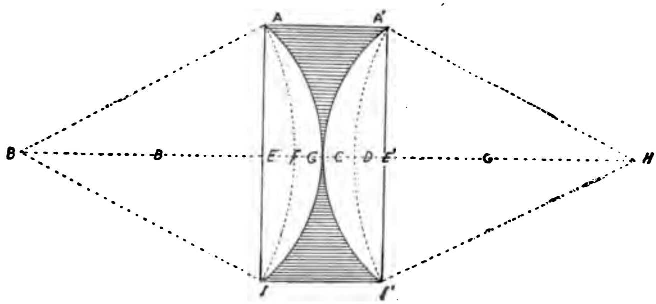
$$\mu = \frac{CG + DF}{CG} = \frac{\left(\frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{R_1} + \frac{1}{R_2}\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}.$$

Substituting $\frac{1}{F}$ for $\frac{1}{f_1} + \frac{1}{f_2}$ (see § 103), we have,

if $R_1 = R_2 = R$,

$$\mu = \frac{\frac{1}{F} + \frac{2}{R}}{\frac{2}{R}} = \frac{R + 2F}{2F} = 1 + \tfrac{1}{2}\frac{R}{F}.$$

(*u*) *Compound Lenses.* Let the lens studied in the last section be cut in two in the plane AI, and the two halves made tangent at G-C, also let the space between the two



halves be filled with a liquid having an index of refraction μ' less than μ. Then if v is the velocity of light in air $v \div \mu$ is its velocity in the lens and $v \div \mu'$ is its velocity in the liquid. The time occupied in passing through the distance $EG + CE'$ is accordingly

$$(EG + CE') \div (\mu \div v) = \mu\,(EG + CE') \div v.$$

The time occupied in passing through an equal distance (from A to A') through the liquid is similarly $\mu'\,(EG + CE') \div v$. The difference between these two times is compensated by the difference in the time required to pass through the distances $BA + A'H$ and $BE + E'H$ in air; that is, the time

required to pass through the distance $EF + DE'$ in air. That is,

$$\mu\,(EG + CE') \div v - \mu'\,(EG + CE') \div v$$
$$= (EF + DE') \div v,$$

whence
$$\mu - \mu' = \frac{EF + DE'}{EG + CE'}.$$

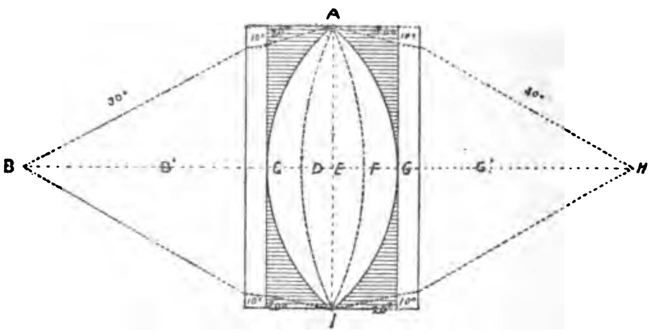
Substituting as in the last section, and cancelling $(AE)^2$ we have

$$EF + DE' = (AE)^2\,(1 \div f_1 + 1 \div f_2) \text{ and}$$
$$EG + CF' = (AE)^2\,(1 \div R_1 + 1 \div R_2)$$

$$\mu - \mu' = \frac{\dfrac{1}{f_1} + \dfrac{1}{f_2}}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \frac{\dfrac{1}{F}}{\dfrac{2}{R}} = \tfrac{1}{2}\,\frac{R}{F}.$$

It follows that

$$\mu' = \mu - \tfrac{1}{2}\,\frac{R}{F}.$$



The same formula holds approximately for a lens mounted between two plates, the spaces being filled with liquid, for

the distances traversed through the lens, the liquid and the air are (nearly) the same.

(*v*) *Electrostatic Potential.* Two bodies, each charged with a unit of positive electricity, repel each other at the distance d with a force (f) such that

$$f = \frac{1}{d^2}.$$

The work (w) required to change the distance between the bodies from d_1 to d_2 is

$$w = f \times (d_1 - d_2) = \frac{d_1 - d_2}{\delta^2},$$

where δ represents some mean between the distances d_1 and d_2.

If we assume that the work W necessary to bring the two bodies together from an infinite distance to the distance d is in general

$$W = \frac{1}{d},$$

we have $W_1 = 1 \div d_1$, $W_2 = 1 \div d_2$, &c., whence by difference

$$w = W_2 - W_1 = \frac{1}{d_2} - \frac{1}{d_1} = \frac{d_1 - d_2}{d_1\, d_2} = \frac{d_1 - d_2}{d^2},$$

where d is the geometric mean between d_1 and d_2. There can evidently be no great error in using the geometric or any other mean when the distances are very small; and by dividing a given motion into a sufficient number of steps the proportional error in the estimation of w can be indefinitely diminished. Now the proportional error in sums cannot be greater than in the separate terms; hence the general formula, $W = 1 \div d$, must be exact. The work W required

to bring a unit of positive electricity to a given point is called the electrostatic potential of that point. We have seen that the electrostatic potential due to one unit of positive electricity at the distance d is $1 \div d$; that due to q units is accordingly

$$e = \frac{q}{d}.$$

When q units are distributed uniformly over the surface of a sphere of radius r, they act upon points outside of the sphere as if they were at the centre of the sphere. The potential of the sphere is determined by the work necessary to bring a unit of positive electricity up to the surface of the sphere, that is, to within a distance r of the charge; hence we have

$$e = \frac{q}{r}, \text{ and } q = er.$$

Let two spheres, suspended as in ¶ 258, be charged to the potential e; then we have

$$q = q' = er = er' = \tfrac{1}{2}ed = \tfrac{1}{2}ed'.$$

The force of repulsion is

$$f = \frac{qq'}{s^2} = \frac{(\frac{1}{2}ed)^2}{s^2} = \frac{1}{4}\frac{e^2d^2}{s^2},$$

where s is the distance between the spheres. This is balanced by a force $w \times g \times \frac{1}{2}s \div l$; hence we have

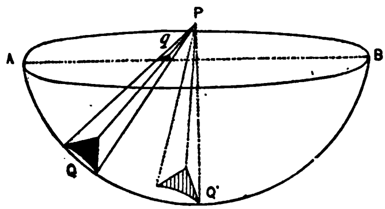
$$\frac{1}{4}\frac{e^2d^2}{s^2} = w \times g \times \tfrac{1}{2}s \div l, \text{ or}$$

$$e^2 = \frac{2wgs^3}{ld^2},$$

whence,

$$e = \sqrt{\frac{2wgs^3}{ld^2}}.$$

(*w*) *Absolute Electrometer.* Let P be a point charged with a unit of positive electricity, and AB an electrified plane near P charged with s units of electricity per unit of surface. Draw the hemisphere $A\,Q\,Q'\,B$, with unit radius and with P as a centre; let



q and Q be sections of the plane and hemisphere included in a small solid angle Q, and let Q' be a similar section such that PQ' is normal to AB. We have, by geometry,

$$\frac{q \cos QPQ'}{(Pq)^2} = \frac{Q'}{(PQ')^2}.$$

Assuming that the hemisphere is also charged with s units of electricity per unit of area, the attractions of q and Q' resolved in the direction PQ' become

$$s \times \frac{q \cos QPQ'}{(Pq)^2}, \text{ and } s \times \frac{Q'}{(PQ')^2}$$

respectively. We have seen that these two quantities into which s is multiplied are equal. Since any two portions of the plate and hemisphere occupying the same solid angle exert the same attraction on the point P, supposing the section of the hemisphere to be transferred to Q', the whole plate must exert the same attraction as the whole hemisphere, neglecting a small portion near the edges, and supposing the whole transferred to Q'. Since the surface of a hemisphere

of unit radius is 2π, and the quantity of electricity is s units per unit of surface, the attraction in question is

$$2\pi s \div (PQ)^2 = 2\pi s.$$

In the absolute electrometer in ¶ 270, we consider the point P to be between two plates with equal and opposite charges of $\pm s$ units per unit of surface. Hence the force (f) or P is $f = 4\pi s$. If the distance between the plates is d, the work required to take P from one to the other is

$$f \times d = 4\pi sd = e,$$

where e is the difference in electrostatic potential. The charge on the upper plate, of area a, is $s \times a$, hence the force F is

$$F = wg = s \times a \times 2\pi s = 2\pi s^2 a.$$

whence $$s = \sqrt{\frac{wg}{2\pi a}}.$$

Substituting this value of s we find

$$e = 4\pi sd = 4\,\pi\sqrt{\frac{wg}{2\pi a}} = \sqrt{\frac{8\pi wg}{a}}.$$

(x) *Capacity of Condensers.* The electrical capacity c of a body is defined as the ratio of the charge (q) to the electromotive force (e); that is the difference of potential which it produces, or by which it is produced. Since, in the case of a sphere of radius r, $q = er$ (see (v)), we have

$$c = \frac{q}{e} = \frac{er}{e} = r. \qquad (1)$$

We have seen in the last section that the difference of potential (e) between two plates charged with $\pm s$ units of elec-

tricity per unit of surface and at a distance d is $4\pi sd$. If the area of either plate is A, the charge is

$$q = \pm\ As,$$

This (q) is the available charge of the condenser formed by the two plates; for a flow of q units from one plate to the other would reduce the charge of each to 0. It follows that the capacity C, or ratio of the charge to the difference of potential or electromotive force is

$$C = \frac{q}{e} = \frac{As}{4\pi sd} = \frac{A}{4\pi d}. \qquad (2)$$

The plates are here supposed to be separated by air, or by other material the specific inductive capacity of which may be taken as unity.

APPENDIX XII.

USEFUL FORMULÆ.

(a) Interpolation.

Let $+x=$ response of instrument to value A; $-y=$ response to value $A+a$; $s=$ sensitiveness $=x+y$,

$$q=A+\frac{xa}{s}. \qquad (\S\ 41)$$

(b) Geometrical Formulæ.

Circumference c of circle of radius r,

$$c=2\pi r. \qquad \text{(Table 3 } F\text{)}$$

Cross-section (q) of cylinder of radius r, weight w, density d, and length l,

$$q=\frac{w}{ld}=\pi r^2; \qquad \text{(Table 3 } G\text{)}$$

whence $$r=\sqrt{\frac{q}{\pi}}=\sqrt{\frac{w}{\pi ld}}.$$

Volume v of sphere of radius r, and diameter d,

$$v=\tfrac{4}{3}\pi r^3=\tfrac{1}{6}\pi d^3; \qquad \text{(Table 3 } H\text{)}$$

whence $$d=1.2407\sqrt[3]{v}.$$

(c) Hydrostatics.

Mass M, Density D, Volume V, and Specific Volume S,

$$S = \frac{1}{D};\quad S = \frac{V}{M};\ V = MS;\ M = \frac{S}{V}; \qquad (\S\ 155)$$

$$D = \frac{M}{V};\ M = VD;\ V = \frac{M}{D}.$$

Pressure p, due to vertical height h, of column of liquid of density d,

$$p = hgd. \qquad (\S\ 63)$$

(d) Expansion.

Reduction of volume V, and density D, of a gas at temperature t, and pressure p, to volume V_0 and density D_0 at 0° and 76 *cm.*,

$$D_0 = D \times \frac{76}{p} \times \frac{273 + t}{273}. \quad \text{(Tables 18, } d\text{–}e\text{; § 81))}$$

$$v_0 = v \times \frac{p}{76} \times \frac{273}{273 + t}. \quad \text{(Tables 18, } f\text{–}g\text{)}$$

Laws of gases, $T,\ T_0,\ T_1,\ T_2$ = absolute temperatures,
$v,\ v_0,\ v_1,\ v_2$ = corresponding volumes,
$p,\ p_0,\ p_1\ p_2$ = corresponding pressures,
(0) at 0°, (1) at 100°, &c.

$$vp = v_0 p_0 = v_1 p_1 = v_2 p_2 \text{ \&c.}$$

(Law of Boyle and Mariotte, § 79)

$$T : T_0 : T_1 : T_2 :: v : v_0 : v_1 : v_2 \text{ \&c.}$$

(Law of Charles, ¶ 76, § 74)

If z = absolute zero,

$$z = -\ 100° \frac{p_0}{p_1 - p_0} = -100° \frac{v_0}{v_1 - v_0} = -\ 273°.$$

(¶¶ 74–76)

Coefficient of expansion e (mean from temperature t_1 to t_2),

$$e = \frac{v_2 - v_1}{v_0\,(t_2 - t_1)}. \qquad (\P\P\ 63\text{–}74)$$

Linear coefficient ϵ (mean relative from t_1 to t_2),

$$\epsilon = \tfrac{1}{3}\, e = \tfrac{1}{3}\, \frac{v_2 - v_1}{v_1\,(t_2 - t_1)}. \qquad (\P\ 240)$$

(e) Calorimetry.

s_1 s_2, &c., = specific heats,
w_1 w_2, &c., = corresponding weights,
t_1 t_2, &c., = corresponding temperatures before mixture,
l_1 l_2, &c., = corresponding latent heats,
c = capacity of calorimeter, t_3 its temperature before mixture, t the temperature of the mixture, and q the no. of units of heat lost,

$$c = w_1 s_1 \times \frac{t_1 - t}{t - t_3}.$$

$$c = w_1 s_1 + w_2 s_2 + w_3 s_3 + \text{\&c.} \qquad (\P\ 91)$$

$$w_1 s_1 (t - t_1) + w_2 s_2 (t - t_2) + c\,(t - t_3) + q + l_1 w_1 = 0. \qquad (\P\ 100)$$

(f) Light.

Law of inverse squares,

$$x : y :: \left(\frac{1}{a}\right)^2 : \left(\frac{1}{b}\right)^2.$$

Photometric law, $x : y :: a^2 : b^2$. (¶ 109)

Principal focal length = F, conjugate focal lengths = f_1 and f_2,

$$F = \frac{1}{\frac{1}{f_1} + \frac{1}{f_2}} = \frac{f_1 \times f_2}{f_1 + f_2}. \quad \text{(For real foci, ¶ 117)}$$

$$F = \frac{1}{\frac{1}{f_1} - \frac{1}{f_2}} = \frac{f_1 \times f_2}{f_2 - f_1}. \quad \text{(For virtual foci, ¶ 119)}$$

$A =$ angle of prism, $D =$ angle of minimum deviation. *Index of refraction,*

$$\mu = \frac{sin\ \frac{1}{2}(A + D)}{sin\ \frac{1}{2}\ A} \qquad \text{(Appendix XI. (}s\text{))}$$

$R =$ mean radius of curvature of double convex lens,
$F =$ principal focal length,

$$\mu = 1 + \frac{1}{2}\ \frac{R}{F}. \qquad \text{(Appendix XI. (}t\text{))}$$

Double convex lens (of index μ) between plates filled with liquid (of index μ'),

$$\mu' = \mu - \frac{1}{2}\ \frac{R}{F}. \qquad \text{(Appendix XI. (}u\text{))}$$

Rotation of plane of polarization (a) of sodium light in degrees due to depth d of sugar solution containing s grams *per cu. cm.*

$$a = 6.65\ d \cdot s \text{ or } s = 0.150\ a \div d. \qquad \text{(¶ 245)}$$

Wave-length l, angle of diffraction a, and distance between lines of grating (d) in position of minimum deviation,

$$l = 2\,d\ sin\ \tfrac{1}{2}a. \qquad \text{(¶ 129)}$$

Correction of observed altitude, A.

$s =$ semidiameter (sun $= 16'$, nearly),
$h =$ dip of horizon (from point m metres high, $1\frac{3}{4}'\sqrt{m}$ nearly),

$r =$ refraction ($1' \times$ cotan A nearly),
$p =$ parallax ($0'$ for the sun),

$$a = A + s - h - r + p. \qquad (\P\ 242)$$

Latitude, l, from altitude a and declination d,

$$l = 90^\circ - a \pm d. \qquad (\P\ 242)$$

Longitude T, in hours, minutes, and seconds, from standard times t' and t'' of equal altitudes, and equation of time e,

$$T = \tfrac{1}{2}(t' + t'') \pm e. \qquad (\P\ 243)$$

(g) Sound.

Lissajous' curves; P, $p =$ pitch; $n =$ No. of lobes, $c =$ No. of cycles per second,

$$P = np \pm c. \qquad (\P\ 143)$$

Velocity v, pitch p, wave-length l, distances traversed d, d_1, d_2, corresponding times, t, t_1, t_2,

$$v = pl\ (\P\ 133) = \frac{d}{t} = \frac{d_1 - d_2}{t_1 - t_2}. \qquad (\P\ 136)$$

$$v = \sqrt{\frac{E}{D}} = \sqrt{\frac{\kappa P}{D}} = \sqrt{\frac{1.41\,P}{D}} \text{ for air.}$$

(Appendix XI. (r))

Velocity (v_1) *of longitudinal vibrations* (Young's modulus $= Y$),

$$v_1 = \sqrt{\frac{Y}{D}}. \qquad (\P\ 248;\ \text{Appendix XI.}\ (r))$$

Velocity (v_2) *of torsional vibrations* (Simple Rigidity $= S$)

$$v_2 = \sqrt{\frac{S}{D}}. \qquad (\P\ 248;\ \text{Appendix XI.}\ (r))$$

Pitch of a string, of length l, and mass $m \times l$, stretched by force f,

$$p = \frac{1}{2}l\sqrt{\frac{f}{m}}. \quad \text{(Appendix VI., 64 } A\text{)}$$

(h) Moments of Inertia.

Moment of inertia of mass M about axis through centre of gravity $= K$; about parallel axis at distance $c = K'$,

$$K' = K + Mc^2. \quad \text{(Appendix XI. } (j)(3))$$

Small mass M at distance l from axis,

$$K = ml^2. \quad \text{(Appendix XI. } (d))$$

Thin ring (or tube) of mass M and mean radius R about its axis (*e. g.* rim of wheel),

$$K = MR^2. \quad \text{(Appendix XI. } (k)\ (1))$$

Thin ring about a diameter (as in spinning),

$$K = \frac{1}{2}MR^2. \quad \text{(Appendix XI. } (k)\ (5))$$

Square bar of length L, breadth B, and mass M, about a central transverse axis (*e. g.* suspended magnet),

$$K = \frac{1}{12}M(L^2 + B^2). \quad \text{(Appendix XI. } (k)\ (4))$$

Round bar of length L, and radius R, about central transverse axis (*e. g.* suspended magnet),

$$K = \frac{1}{12}ML^2 + \frac{1}{4}MR^2. \quad \text{(Appendix XI. } (k)\ (10))$$

Disc or cylinder of mass M and radius R about axis (*e. g.* a wheel),

$$K = \frac{1}{2}MR^2. \quad \text{(Appendix XI. } (k)\ (7))$$

Thin disc about diameter (*e.g.* a coin spinning),

$$K = \tfrac{1}{4} MR^2. \quad \text{(Appendix XI. }(k)\ (8))$$

Sphere of mass M and radius R,

$$K = \tfrac{2}{5} MR^2. \quad \text{(Appendix XI. }(k)\ (12))$$

(l) Dynamics.

Force f, acting for time t, gives mass m velocity v,

$$ft = mv. \qquad (\S\ 106)$$

$$f = \frac{mv}{t}\ ;\ t = \frac{mv}{f};\ m = \frac{ft}{v}\ ;\ v = \frac{ft}{m}.$$

Gravity (g), time t, velocity v, distance d,

$$v = gt. \qquad (\S\ 108)$$

$$d = \tfrac{1}{2} gt^2. \qquad (\S\ 108)$$

Ballistic pendulum,

$$v = AB\sqrt{\frac{g}{AC}}. \qquad (\S\ 109)$$

Pendulum of length l, time t (latitude λ),

$$t = \pi\sqrt{\frac{l}{g}}. \quad \text{(Appendix XI. }(c)\ (2)).$$

$$l = \frac{gt^2}{\pi^2}. \quad \text{(Appendix XI. }(c)\ (3))$$

$$l = 99.3562 - 0.2536 \cos 2\lambda. \quad \text{(Table 48)}$$

$$g = \frac{\pi^2 l}{t^2}. \quad \text{(Appendix XI. }(c)\ (1))$$

$$g = 980.6056 - 2.5028 \cos 2\lambda. \quad \text{(Table 47)}$$

Compound pendulum with directive force D, and moment of inertia K,

$$t = \pi \sqrt{\frac{K}{D}}. \quad \text{(Appendix XI. (}d\text{) (2))}$$

Directive force D of magnet of moment M due to horizontal component H of earth's magnetic field,

$$D = MH = \pi^2 \frac{K}{t^2}. \quad \text{(Appendix XI. (}d\text{) (1))}$$

(j) Elasticity.

Coefficient of torsion T,

$$T = \frac{D}{57^\circ.3} = \frac{\pi^3 K}{180\, t^2}. \quad \text{(Appendix XI. (}g\text{) (2))}$$

Simple rigidity S, of a wire of length l, radius r, and coefficient of torsion T,

$$S = \frac{360\, Tl}{\pi^2 r^4} = \frac{2\pi l K}{r^4 t^2}.$$

(Appendix XI. (o) (3) (4))

Young's modulus Y, for a beam of length l, breadth b, thickness t, suffering deflection d, from force f at middle of beam.

$$Y = \frac{f l^3}{4 b d t^3} = \tfrac{1}{4} F. \quad \text{(¶ 163; Appendix XI. (}p\text{))}$$

Resilience of volume, with coefficient or modulus M,

$$M = \frac{SY}{9S - 3Y}.$$

(¶ 240; Appendix XI. (m) (1))

Poisson's ratio (μ) of lateral contraction to longitudinal extension,

$$\mu = \frac{3M - 2S}{6M + 2S}. \quad \text{(Appendix XI. } (m) \text{ (2))}$$

(k) Friction.

Coefficient of friction in fluids f, creating force F on area a through velocity v,

$$f = \frac{F}{av^2}. \qquad (\P\ 172)$$

Viscosity coefficient η, in capillary tube of length l and radius r, transmitting in time t, a weight w, of liquid of density d, under a pressure $p = hgd$,

$$\eta = \frac{\pi g d^2 h r^4 t}{8\,wl}. \quad (\P\ 251\text{; Appendix XI. } (l))$$

Efficiency e of water motor with wheel of circumference c making n revolutions per unit of time against tangential force f, while consuming a volume v of water under the pressure p,

$$e = \frac{cnf}{vp}. \qquad (\P\ 175)$$

Mechanical equivalent of heat J, in terms of number of times (n) that a material of specific heat s must fall through a distance d, under gravity (g) to warm itself t°,

$$J = \frac{ndg}{st}. \qquad (\P\ 178).$$

(l) Magnetism.

Mean strength s, of the poles of two parallel magnets, the attraction of which at the distance d is greater than the repulsion by amount Δ,

$$s = \tfrac{1}{2}\, d \sqrt{\Delta} \text{ (nearly)}. \qquad (\S\ 129)$$

Moment of magnet with poles of strength $\pm s$ and distance l between poles,

$$M = s \times l. \qquad (\P\ 185)$$

Magnetic couple (c) deflecting a wire of coefficient of torsion T, a°, or giving body with moment of inertia K, a time of vibration t, in earth's horizontal field H,

$$MH = Ta\,(\P\ 182) = \frac{\pi^2 K}{t^2}. \qquad (\text{Appendix XI. } (h))$$

Maximum deflection a, due to magnet of moment M at mean distance d, in earth's horizontal field H,

$$\frac{M}{H} = \tfrac{1}{2}\, d^3 \tan a. \qquad (\text{Appendix XI. } (i)\ (1)$$

Horizontal intensity H of earth's magnetism,

$$H = \frac{\pi}{t}\sqrt{\frac{2K}{d^3 \tan a}}. \qquad (\text{Appendix XI. } (i)\ (2))$$

Dip (d), estimated by throws of ballistic galvanometer; a' due to vertical, a'' due to horizontal components,

$$\tan d = \frac{\text{chord } a'}{\text{chord } a''}. \qquad (\P\ 101)$$

(m) Magnetic Current Measure.

Constant K of a coil with n turns of radius r,

$$K = \frac{2\pi n}{r}. \qquad (\P\ 199,\ \S\ 133)$$

Reduction factor of galvanometer with constant K in magnetic field H, deflected a° by current C,

$$i = \frac{H}{K} = \frac{c}{\tan a} \text{ for absolute units,}$$

$$I = 10\frac{H}{K} = \frac{C}{\tan a} \text{ for ampères.} \qquad (\P\ 190)$$

Comparison of tangent galvanometers with reduction factors I and I', giving deflections a and a',

$$\frac{I}{I'} = \frac{\tan a}{\tan a'}. \qquad (\P\ 201)$$

Shunt of resistance S increases reduction factor of galvanometer of resistance $R + G$ in the ratio

$$\frac{i}{I} = \frac{S}{R + G + S}. \text{ (Appendix VIII., 61 } C\text{).}$$

Dynamometer with large coil of constant K, and small coil of magnetic area A, gives deflection a, under current in ampères C, against torsion of wire having coefficient t, such that

$$C = 10\sqrt{\frac{ta}{KA}}. \qquad (\P\ 204)$$

Electro-chemical current measure in terms of weight w, of substance having electro-chemical equivalent q, acted upon in time t,

$$c = \frac{w}{qt}.$$

For copper,

$$C = \frac{3050\, w}{t} \text{ ampères.} \qquad (\P\ 206\ (2))$$

Current C, of heat or of electricity in terms of quantity Q, in time t,

$$C = \frac{Q}{t} \text{ (by definition).}$$

Specific conductivity ς, in terms of current C, length of conductor L, area of its cross-section A, and difference of potential or temperature, E or T,

$$\varsigma = \frac{CL}{AT} = \frac{CL}{AE}. \qquad (¶\ 241)$$

(n) Electrical Resistance.

Resistance R, of conductor, in which a current C in ampères, generates heat enough in the time T, to raise a weight w of water, and a calorimeter of thermal capacity c, from $t_1°$ to $t_2°$,

$$R = \frac{4.17\ (w + c)\ (t_2 - t_1)}{C^2\ T} \qquad (¶\ 213)$$

Specific resistance S, of conductor of length L, cross-section A, and resistance R,

$$S = \frac{RA}{L}. \qquad (¶\ 219)$$

Wheatstone's Bridge (see Fig. 18, page 732),

$$AB : BC :: AD : DC. \qquad (§\ 141)$$

Resistance (R) *in multiple arc* of conductors having resistances R_1 R_2, &c.,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \text{\&c.} \qquad (§\ 140)$$

Thomson's Method. Battery resistance B, galvanometer resistance G; if external resistance R gives same deflection as r gives with battery shunt of resistance S,

$$B = S\frac{R - r}{r + G}. \quad \text{(Appendix VI., 113 } A\text{)}$$

Ohm's Method. Battery resistance B, electromotive force E, deflection a; with added resistance R_2, deflection a_2,

$$B = \frac{R_2 \tan a_2}{\tan a_1 - \tan a_2}. \qquad (\P\ 225)$$

$$E = \frac{R_2 \tan a_1 \tan a_2}{\tan a_1 - \tan a_2}. \qquad (\P\ 230)$$

Beetz' Method. Resistance of stronger battery B, electromotive force E', external resistance r_1' and r_2', corresponding resistances of shunt r_1 and r_2,

$$B = \frac{r_1 r_2' - r_1' r_2}{r_2 - r_1}. \qquad (\P\ 228)$$

$$\frac{E'}{E''} = \frac{B + r_1 + r_1'}{r_1} = \frac{B + r_2 + r_2'}{r_2}. \qquad (\P\ 228)$$

(o) Electromotive Force.

Electro-chemical equivalent q, heat of combustion h, electromotive force E, and mechanical equivalent J,

$$E = Jqh. \qquad (\S\ 145)$$

Electrical power (P) in terms of electromotive force E, and current C,

$$P = CE = C^2R; \qquad (\P\ 230;\ \S\S\ 136,\ 137)$$

whence $C = P \div E$, and $E = P \div C$.

Electromotive force in terms of the current C and resistance R,

$$E = CR; \qquad (\S\ 139)$$

Whence *Ohm's Law*,

$$C = \frac{E}{R}. \qquad (\S\ 138)$$

Wiedemann's Method. Electromotive forces E and e in conjunction and opposition, corresponding deflections A and a,

$$\frac{E}{e} = \frac{tan\ A + tan\ a}{tan\ A - tan\ a}. \qquad (\P\ 231)$$

Electromotive forces E and e produce equal currents with given external resistances; also with the resistances R and r added to respective circuits; then

$$e : E :: r : R. \qquad (\P\ 233)$$

Differences of Potential e_1 and e_2, corresponding to distances d_1 and d_2 on uniform straight wire carrying a current,

$$e_1 : e_2 :: d_1 : d_2. \qquad (\P\P\ 235, 236)$$

(p) Electrostatics.

Capacity c of sphere of radius r,

$$c = r. \qquad \text{(Appendix XI. } (x)\ (1))$$

Capacity of condenser with insulating layers of area A, thickness t, and specific inductive capacity s,

$$c = \frac{As}{4\pi t}. \qquad \text{(Appendix XI. } (x)\ (2))$$

Charge q, in condenser of capacity c, due to electromotive force e (by definition),

$$q = ce.$$

Electromotive force e in electrostatic measure causing two pith-balls of diameter d, weight wg, suspended by cords of length l, to diverge through distance s,

$$e = \sqrt{\frac{2wgs^3}{ld^2}}. \qquad \text{(Appendix XI. } (v))$$

Electromotive force e in electrostatic measure causing a plate of area a to be attracted or repelled by a large plate at a distance d, with a force wg,

$$e = d\sqrt{\frac{8\pi g w}{a}}. \qquad \text{(Appendix XI. } (w))$$

(q) Average.

$$o \overline{}_{x^n} a = \frac{a^n}{n+1}. \qquad \text{(Appendix IX. } (i))$$

(r) Probable Error.

$P =$ probable error of single observations,
$p =$ probable error of mean of n observations,
$d^2 =$ mean square of the differences,

(Appendix X. (k))

$$P = 0.67449\, d\sqrt{1 \div (n-1)} = p\sqrt{n}.$$

Probable error (p) of a result, in terms of variations d_1 d_2, &c., introduced by changing the separate data by an amount equal to probable error of each,

$$p = \sqrt{d_1^2 + d_2^2 + \&c.} \qquad \text{(Appendix X. } (o))$$

(s) Weight of Results.

Weights $= w_1\ w_2\ w_3$, &c. (Appendix X. (s))
probable errors $= p_1, p_2, p_3$, &c.

$$\text{then } w_1 : w_2 : w_3, \&c., :: \left(\frac{1}{p_1}\right)^2 : \left(\frac{1}{p_2}\right)^2 : \left(\frac{1}{p_3}\right)^2 \&c.$$

Most probable result R, in terms of several results, $r_1\ r_2\ r_3$, &c., with weights $w_1\ w_2\ w_3$, &c.,

$$R = \frac{w_1 r_1 + w_2 r_2 + w_3 r_3 + \&c.}{w_1 + w_2 + w_3 + \&c.}.$$

(t) Dimensions.

NOTE. The dimensions of a quantity may be defined as a mathematical expression for the number of times that multiples of the three fundamental units of length (L) mass (M) and time (T) must be employed as factors to express the quantity in question. The dimensions are usually represented by ordinary "exponents."[1]

Dimensions are useful in reducing results from one system to another. Let L be the value in centimetres of the unit of length in any system, M the value in grams of the unit of mass, T the value in seconds of the unit of time; then the dimensions of a given quantity, let us say $L^x M^y T^z$, give at once the factor for reducing that quantity from the given system to *C. G. S.* units.

Dimensions obey the following laws: —

(1) Only quantities of a given kind can be added or subtracted, and the sum has the same dimensions as the separate quantities.

(2) The dimensions of the product or quotient of two quantities are equal to the product or quotient of their separate dimensions treated as algebraic quantities. It is through this law that dimensions are calculated.

(3) The two sides of an equation must always have the same dimensions; for quantities differing no matter how slightly in dimensions are, like surfaces and volumes, essentially different in kind, and hence cannot be numerically or quantitatively compared. This equality of dimensions, being a condition which every rational formula must satisfy, furnishes a useful test of the accuracy of mathematical work.

Angles, strains, specific gravity, temperature, and all *rela-*

[1] For proofs and illustrations, see Kohlrausch, Physical Measurement, Appendix A.

tive magnitudes, having no dependence upon the fundamental units, are of dimensions 0.

The dimensions of other quantities are expressed as follows:

Length	L
Surface	L^2
Volume	L^3
Time	T
Velocity	$L \div T$ or LT^{-1}
Acceleration	$(L \div T) \div T$ or LT^{-2}
Mass	M
Density	$L^{-3}M$
Force	LMT^{-2}
Work (or kinetic energy)	L^2MT^{-2}
Couple	L^2MT^{-2}
Directive Force	L^2MT^{-2}
Power	L^2MT^{-3}
Moment of inertia	L^2M
Stress, or pressure	$L^{-1}MT^{-2}$
Modulus of elasticity	$L^{-1}MT^{-2}$
Electrostatic or magnetic unit	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-1}$
Electrostatic potential	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Electrostatic capacity	L
Magnetic moment	$L^{\frac{5}{2}}M^{\frac{1}{2}}T^{-1}$
Magnetic field	$L^{-\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Electrical current (magnetic measure)	$L^{\frac{1}{2}}M^{\frac{1}{2}}T^{-1}$
Electro-magnetic unit of quantity	$L^{\frac{1}{2}}M^{\frac{1}{2}}$
Electromotive force	$L^{\frac{3}{2}}M^{\frac{1}{2}}T^{-2}$
Electro-magnetic capacity	$L^{-1}T^2$
Resistance	LT^{-1}

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ABBREVIATIONS.

app., apparatus.
B. W. G., Birmingham wire gauge.
C. G. S., centimetre-gram-second system.
cm., centimetre [s].
coef., coefficient [of].
const., constant.
cu., cubic.
det., determination [of].
e. m. f., electromotive force.
eq., equivalent.
ex., exercise.
exp., experiment.
g., gram [s]; also acceleration of gravity.
gr., gravity; also grain.
h., hour [s].
H. U., Harvard University.
kilo., kilogram [s].
kilom., kilometre [s].
m., metre [s]; also minute [s].
meas., measurement [of].
min., minute [s].
mm., millimetre [s].
obd., observed.
obs., observation [s].
s. or *sec.*, second [s].
sp., specific.
sp. gr., specific gravity.
sp. ht., specific heat.
sq., square.
temp., temperature.
vol., volume.
wt., weight.

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